POMS PRODUCTION AND OPERATIONS MANAGEMENT

PRODUCTION AND OPERATIONS MANAGEMENT POMS
Vol. 23, No. 12, December 2014, pp. 2152–2162
ISSN 1059-1478 | EISSN 1937-5956 | 14 | 2312 | 2152
DOI 10.1111/poms.12251

Quality Risk Ratings in Global Supply Chains

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Extended enterprises face many challenges in managing the product quality of their suppliers. Consequently characterizing the quality risk posed by value-chain partners has become increasingly important. There have been several recent efforts to develop frameworks for rating the quality risk posed by suppliers. We develop an analytical model to examine the impact of such quality ratings on suppliers, manufacturers, and social welfare. While it might seem that quality ratings would benefit high-quality suppliers and hurt low-quality suppliers, we show that this is not always the case. We find that such quality ratings can hurt both types of suppliers or benefit both, depending on the market conditions. We also find that quality ratings do not always benefit the most demanding manufacturers who desire high-quality suppliers. Finally, we find that social welfare is not always improved by risk ratings. These results suggest that public policy initiatives addressing risk ratings must be carefully considered.

Key words: quality risk; vendor rating; supplier rating and evaluation; global supply chain; analytical modeling
History: Received: January 2012; Accepted: January 2014 by Haresh Gurnani, Saibal Ray, and Yunzeng Wang, after 2 revisions.

1. Introduction

The globalization of economies expands the boundaries of extended enterprises. Dell Inc. directly or indirectly manages over 100 key suppliers in over 15 countries (Dell 2013). Toyota UK has 260 parts and materials suppliers based in the UK and Europe (ToyotaUK 2013). Global extended enterprises such as Dell and Toyota face incredible complexity. Among many supply chain challenges, managing the product quality risk posed by such far-flung suppliers has become increasingly difficult. In 2006, Dell announced a recall of its notebook batteries due to fire hazard (CPSC News 2006). More than 4.1 million battery packs were deemed unsafe and recalled. Sony, the supplier of these battery packs, also sold laptop batteries to HP and Toshiba. In 2008, Sony batteries were figured again as the culprit of laptops overheating. About 100,000 Sony laptop batteries were recalled in that year (Ogg 2008).

Toyota recalled over 12 million vehicles because of claims of sudden acceleration (Welch 2011). The recall was linked to the CTS Corporation, an Indiana-based automotive supplier that makes the gas-pedals assemblies for Toyota. CST not only makes gas pedals for Toyota and Chrysler, but also for Honda, Ford, and General Motor. Chrysler was also forced to recall 35,000 Dodge and Jeep models for sticky gas pedals made by CTS Corporation (Hirsch 2010).

As manufacturers' source from suppliers spread around the world, the risks posed by those suppliers have grown. From lead paint in Mattel toys to melamine contamination of Cadbury Chocolates, vendor quality failures have touched nearly every industry. In many cases, the quality risk was not well managed or understood due to low supply chain visibility. The quality risk posed by a supplier is largely determined by how much quality risk information is disclosed. Such quality risk information includes sourcing practices, quality practices, management capabilities, business maturity, financial stability, quality inspection data, and quality audits. If a manufacturer is located far from its supplier, it can be difficult for the manufacturer to obtain *reliable* quality risk information. Moreover, quality problems may be caused by multi-tier sourcing (i.e., a manufacturer's supplier sources from a supplier's supplier in a lower-tier of the supply chain). Multi-tier sourcing increases the length of the supply chain and increases the risk. It reduces the accountability of product quality because it is hard to track the quality control process of the whole supply chain. To ensure supply reliability and thus reduce disruption risk, a manufacturer may source the same materials from different suppliers. However, this too increases quality risk by introducing more failure points.

To mitigate quality risk, researchers have proposed frameworks to manage quality risk throughout global supply chains, including the adoption of supplier ratings. Tse and Tan (2011) proposed measuring product quality risk in four categories: risk from supplier's production, supplier's quality standard, risk from supplier's logistics service, and product quality risk visibility. Van Weele (2009) suggested that a manufacturer assess its supplier's product quality at four levels: product level, process level, quality assurance system level, and company level. The product level focuses on inspecting incoming products and establishing the quality level of incoming products. The process level examines whether the quality procedures in production are under control. The quality assurance system level checks the way in which procedures regarding quality inspection are developed, kept up-to-date, maintained, and refined. The company level considers both quality aspects and financial performance aspects.

Professional quality risk rating services, focused on global manufacturing suppliers, are offered by a number of firms such as Trivista and Asia Inspection. Some industries are making efforts to develop common quality risk assessments. The Quality Certification Alliance (QCA), which was formed in 2008 by major promotional product sellers, aims to "elevate the standards by which industry firms that import and/or manufacture promotional products provide consistently safe, high-quality, socially compliant and environmentally conscientious merchandise" (QCA 2011). Service industries also face similar vendor risk. For example, in the financial industry, major banks have been developing a common information risk assessment to rate the risk posed by vendors (Johnson et al. 2009). In both manufacturing and services, many large customers often source from a limited number of suppliers (e.g., CTS selling gas pedals to Toyota, Honda, GM, and Chrysler). Thus a shared quality risk assessment helps both suppliers and their customers avoid redundant and repeated assessments.

Note that the professional quality risk assessments or ratings that we consider in this study are not simply certifications. For example, the ISO 9000 series of certifications have been widely adopted throughout the world (Albuquerque et al. 2007). Such certifications both improve quality and provide quality signals for customers. However, such binary measures do not provide an ongoing measure of quality risk. In this study, we consider a professional risk rating that is more similar to a bond rating, where risk is accessed and updated on a regular basis. Moreover, the risk we consider is a quality failure that causes a significant financial loss for the manufacturer such as a recall.

Given the similarities, it is tempting to equate quality risk rating with ratings of financial instruments. However, quality ratings are quite different from bond or credit ratings (which measure the default probability for a debt issuer). A good credit rating generally enables the debt issuer to raise money from financial markets at a lower cost (Kliger and Sarig 2000). However, a good quality rating may not directly benefit a high-quality supplier because the quality rating may have subtle impacts on the competition among suppliers, the incentives to improve quality levels and reduce the risk of recall, and the prices charged to manufacturers. In this study, we focus on the following research questions:

- Does quality risk rating always benefit the high-quality supplier (or hurt the low-quality supplier), as the prior literature on finance predicts? If not, how does quality rating affect different suppliers under different market conditions?
- Does quality risk rating always benefit the most demanding manufacturers who desire high-quality business partners?
- Does quality risk rating increase social welfare?

We develop an analytical model to examine the impact of quality risk ratings on suppliers, manufacturers, and social welfare. We do this by comparing two cases: (1) the case where a professional quality risk rating is provided (e.g., by QCA), and (2) the case where manufacturers perform assessments by themselves. We find that quality ratings can hurt or benefit both types of suppliers, depending on the market conditions. Likewise, our analysis leads to another result: quality ratings can hurt demanding manufacturers. Prior results in the licensing literature showed that improved information always benefits the high-needs manufacturers at the cost of less demanding manufacturers (Shapiro 1986). We show a different result in this study.

We begin by examining the related literature. Then we present our model, considering two types of suppliers (low and high quality) and two types of manufacturers (those who place low and high value on quality). We analyze the model for the case with professional quality ratings in section 3, then the model for the case with manufacturer assessments in section 4. We compare the two cases in section 5. Finally, we conclude with recommendations for researchers and policy makers.

1.1. Literature Review

The economics literature has long studied information revelation and signaling of quality. Akerlof (1970) pointed out that a lack of quality information can lead to market failures. Viscusi (1978) examined a process of sequentially revealing the quality of firms and gave maximum prices that firms are willing to pay for having their quality revealed. Leland (1979) suggested that introducing a minimum quality standard, or "licensing" standard can increase social welfare. Shapiro (1983) examined a market where buyers know product quality some time after the purchase. He showed that a firm's cost of establishing its reputation should be covered by gains from its established reputation. Shapiro (1986) showed that licensing and certification tend to benefit consumers who value quality highly at the expense of those who do not. Lizzeri (1999) examined strategic quality information revelation by certification intermediaries. He provided conditions under which "certification as a minimal quality standard" is an optimal choice of a mopolist certification intermediary. He also showed that competition among intermediaries can lead to full information revelation.

This stream of literature generally assumed that quality is exogenously given (Akerlof 1970, Leland 1979, Lizzeri 1999, Viscusi 1978). That is, these studies ignored how sellers (those to be certified) respond to quality. In contrast, the quality levels of suppliers are endogenously determined in our study. Quality ratings not only reveal information, but also influence the incentive of the suppliers' efforts on quality. The other difference between our study and the prior literature is that our study considers duopoly competition while prior literature focused on a fully competitive market (Shapiro, 1983, 1986).

Another stream of supply chain literature has studied managing supply disruption risk in global supply chains. Gümüş et al. (2012) considered the risk of supply disruption in a supply chain with a single buyer, a reliable and expensive supplier, and a cheaper but less reliable supplier. They showed that the unreliable supplier may use a price and quantity (P&Q) guarantee contract to better compete against the more reliable one by providing supply assurance to the buyer. When information asymmetry risk is high, the P&Q guarantee contract enables the unreliable supplier to credibly signal her true risk, thereby improving visibility in the chain. Gurnani and Shi (2006) considered a bargaining game between a buyer and a supplier who have different estimates about supply reliability. They computed the optimal contract P&Q, and discuss the role of using down-payment or nondelivery penalties in the contract. Yang et al. (2009) used mechanism design theory to design an optimal contract menu offered by a manufacturer to its supplier, who has private information about supply disruption. They showed that the less reliable supplier chooses to stop using backup production while the more reliable supplier continues to use it. Subsequently, contract choice by the supplier reveals its private information about supplier disruption.

In contrast to this literature that focuses on supply disruption, this study focuses on managing supplier quality risk. Huang et al. (2006) compare vendor certification with vendor appraisal in a supply chain with a single supplier and a single manufacturer. We focus on quality risk ratings in a supply chain with competing suppliers.

2. The Model

We adopt a vertical differentiation framework (see, e.g., Bhargava and Choudhary 2008) for manufacturers who have different usage utilities for products sourced from suppliers. We model two risk-neutral *representative* manufacturers sourcing a single product (or a single batch of products)[1] from their suppliers: (1) low-type manufacturer (Manufacturer L), whose usage utility from the product sourcing from its supplier is V with $V > 0$, and (2) high-type manufacturer (Manufacturer H), whose usage utility from product sourcing from its supplier is θV with $\theta > 1$.

A supplier exerts effort e ($e \sim [0,1]$) to increase the quality level of its product and reduce its risk to the manufacturer. We normalize the supplier risk, the probability that the supplier's product creates a recall, to $1 - e$. That is, when the supplier exerts greater effort, it is less likely to cause a recall by the manufacturer. The cost of exerting effort e is assumed to be a convex function: ce^2, where $c > 0$ is the quality cost parameter. When the supplier's product fails, the manufacturer incurs a loss proportional to its usage utility. We use λV and $\lambda\theta V$ to denote the loss of the low-type manufacturer and the high-type manufacturer, respectively. In general, manufacturers do not source from suppliers with a very high risk. Therefore, we assume that λ, the proportional loss at a recall, satisfies $0 < \lambda < 1/2$. The effort of the lower-quality supplier (Supplier L) is denoted by e_l while the effort of the higher-quality supplier (Supplier H) is denoted by e_h ($e_l \le e_h$).

The two suppliers engage in a two-period competition. In Period 0, the suppliers know whether they will be rated before Period 1 and then determine their quality levels sequentially. Supplier H is a leader while Supplier L is a follower. In Period 1 and Period 2, both the suppliers choose their pricing strategies and then sell their products to manufacturers. If a professional rating is provided before Period 1, the manufacturers will know the quality risk levels of both suppliers (see Figure 1). However, if a professional rating is not provided before Period 1, then the risk levels of both suppliers are unobservable to manufacturers in Period 1. Thus, both suppliers *appear* to be the same to manufacturers in Period 1. However, suppliers cannot hide their quality levels forever. In time, the manufacturers will eventually know the

Figure 1 Sequence of Events When a Professional Rating is Provided

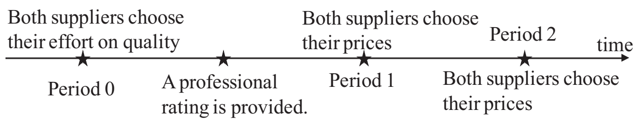
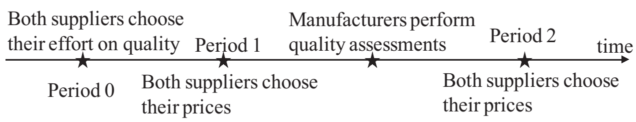


Figure 2 Sequence of Events When a Professional Rating is not Provided



suppliers' quality risk levels before Period 2 via the manufacturers' own assessments (see Figure 2). This means that the professional quality rating agencies are more efficient than individual manufacturers.

We assume that suppliers use a "price matching" marketing strategy to prevent a Bertrand competition, which drives profits to zero for both suppliers. (Zhang 1995). The "price matching" strategy avoids a head-to-head competition by making it impossible for firms to steal each other's customers by simply cutting prices. Our model differs from the Bertrand competition where firms set their prices only once. We further assume that manufacturers do not know who is the first-mover or the second-mover. Otherwise, manufacturers may use that information to infer the quality levels of suppliers. This assumption is reasonable when both suppliers enter the markets at nearly the same time.

For ease of exposition, we use p_l and p_h to denote lower and higher prices charged by suppliers. π_l and π_h denote the total profit of lower- and higher-quality suppliers in both periods. Next we analyze two cases: (1) when a quality rating is provided in Period 1, and (2) when manufacturers perform their own risk assessments (Table 1).

3. Competition with Quality Ratings Provided

Quality ratings reveal the quality levels of suppliers to manufacturers in Period 1. Hence in this case, manufacturers know e_l and e_h in both periods. The competition in Period 1 is the same as that in Period 2. Hence, in Period 2, a supplier charges the same price as that charged in Period 1; a manufacturer chooses the same supplier as that chosen in Period 1. Thus, we only need to focus on a single period.

We use U_{tq} to denote the net surplus of a type-t manufacturer who uses a supplier with a quality level of q, where $t = L$ (low-type manufacturer) or H (high-type manufacturer); $q = l$ (lower quality level) or h

Table 1 Table of Notation

V	Low-type manufacturer's usage utility
θV	High-type manufacturer's usage utility
λ	Proportional loss of a manufacturer's utility when its supplier's product fails
c	Quality cost parameter
e_h	Effort of high-quality supplier
e_l	Effort of low-quality supplier
p_h	Price of high-quality supplier
p_l	Price of low-quality supplier
p_i	Introductory price of both suppliers in Period 1 when quality risk ratings are not provided
π_h	Profit of high-quality supplier
π_l	Profit of low-quality supplier
U_{tq}	Net surplus of a type-t manufacturer who chooses a supplier with quality of q
U_{tr}	Net surplus of a type-t manufacturer who randomly chooses a supplier
Case NR	The case where quality risk ratings are not provided in Period 1
Case R	The case where quality risk ratings are provided in Period 1

(higher quality level). The expressions of U_{tq} are as follows.

$$U_{Ll} = e_l(V - p_l) + (1 - e_l)[(1 - \lambda)V - p_l]$$
$$U_{Lh} = e_h(V - p_h) + (1 - e_h)[(1 - \lambda)V - p_h]$$
$$U_{Hl} = e_l(\theta V - p_l) + (1 - e_l)[(1 - \lambda)\theta V - p_l]$$
$$U_{Hh} = e_h(\theta V - p_h) + (1 - e_h)[(1 - \lambda)\theta V - p_h]$$

We consider the pricing strategies of both manufacturers. If both suppliers stay in the market, then given e_l, e_h and $e_h > e_l$, there are three possible scenarios in equilibrium: (A1) Supplier H sells to Manufacturer H while Supplier L sells to Manufacturer L, (A2) Supplier H sells to Manufacturer H and L whereas Supplier L sells Manufacturer L, (A3) Both Supplier H and Supplier L sell to Manufacturer H.

In Case A1, Supplier H chooses a price-matching policy to ensure that $U_{Hh} = U_{Hl} + \varepsilon > 0$ ($\varepsilon > 0$, $\varepsilon \to 0$). Given Supplier H's price matching policy, Supplier L can never poach any customer from Supplier H (even though it sets $p_l = 0$). Thus, Supplier L will choose p_l such that $U_{Ll} = 0$ to maximize its profit. In Case A2, Supplier H's price-matching policy satisfies U_{Lh}=U_{Ll}. Supplier L will set p_l such that $U_{Ll} = 0$. In Case A3, Supplier L chooses p_l such that $U_{Hl} = 0$. Supplier H chooses a price-matching policy such that $U_{Hh} = U_{Hl}$. Next, we will show conditions under which Case A1, A2 or A3 occurs in equilibrium.

The equilibrium prices in both periods only depend on e_h and e_l chosen in Period 0. Thus, the game in Period 2 is the same as that in Period 1. And the equilibrium prices in Period 2 are the same as those in Period 1. In Case A1, we obtain p_l and p_h by solving $U_{Hh} = U_{Hl}$ and $U_{Ll} = 0$.

$$p_{l1} = V(1 - \lambda + \lambda e_l), \tag{1}$$

$$p_{h1} = V\theta\lambda(e_h - e_l) + V(1 - \lambda + \lambda e_l). \tag{2}$$

The low-quality supplier obtains a revenue of p_{l1} from the low-type manufacturer in each period and thus, its total revenue is $2p_{l1}$. Likewise, the high-quality supplier obtains a total revenue of $2p_{h1}$ from the high-type manufacturer. The total profits of the high-quality supplier and low-quality supplier in two periods can be written as follows:

$$\pi_{l1} = 2p_{l1} - ce_l^2 = 2V(1 - \lambda + \lambda e_l) - ce_l^2, \tag{3}$$

$$\begin{aligned}\pi_{h1} &= 2p_{h1} - ce_h^2 \\ &= 2V\theta\lambda(e_h - e_l) + 2V(1 - \lambda + \lambda e_l) - ce_h^2.\end{aligned} \tag{4}$$

In Case A2, we may obtain p_l and p_h by solving $U_{Lh} = U_{Ll}$ and $U_{Ll} = 0$.

$$p_{l2} = V(1 - \lambda + \lambda e_l), \tag{5}$$

$$p_{h2} = V(1 - \lambda + \lambda e_h) \tag{6}$$

The low-type manufacturer gets the same net surplus from either buying from Supplier L or buying from Supplier H. Thus, the low-type manufacturer buys from either Supplier L or Supplier H with a probability of 50% for each. When $e_h > e_l$, the high-type manufacturer gets higher net surplus from Supplier H than from Supplier L because $(U_{Hh} - U_{Hl})|_{p_h=p_{h3},p_l=p_{l3}} = V\lambda(e_h - e_l)(\theta - 1) > 0$. Thus, the high-type manufacturer buys from Supplier H in both periods. It follows that the demand for Supplier H is 3 while the demand for Supplier L is 1 in both periods. The total profits of the high-quality supplier and low-quality supplier in two periods are

$$\pi_{l2} = p_{l2} - ce_l^2 = V(1 - \lambda + \lambda e_l) - ce_l^2, \tag{7}$$

$$\pi_{h2} = 3p_{h2} - ce_h^2 = 3V(1 - \lambda + \lambda e_h) - ce_h^2 \tag{8}$$

Now, consider Case A3, where p_l and p_h can be obtained by solving $U_{Hh} = U_{Hl}$ and $U_{Hl} = 0$.

$$p_{l3} = V\theta(1 - \lambda + \lambda e_l), \tag{9}$$

$$p_{h3} = V\theta(1 - \lambda + \lambda e_h). \tag{10}$$

It can be verified that the low-type manufacturer gets negative net surplus no matter if it buys from Supplier L or Supplier H. Thus, the low-type manufacturer does not buy from any supplier. Since the high-type manufacturer gets the same net surplus from both suppliers, it will randomly choose a supplier with a probability of 50% for each. Thus, the low-quality supplier obtains a revenue of $\frac{1}{2}p_{l2}$ from the high-type manufacturer in each period and thus, its total revenue is p_{l2} . Similarly, the high-quality supplier obtains a total revenue of p_{h2} from the high-type manufacturer. The total profits of the high-quality supplier and low-quality supplier in two periods can be written as

$$\pi_{l3} = p_{l3} - ce_l^2 = V\theta(1 - \lambda + \lambda e_l) - ce_l^2, \tag{11}$$

$$\pi_{h3} = p_{h3} - ce_h^2 = V\theta(1 - \lambda + \lambda e_h) - ce_h^2. \tag{12}$$

When $e_h = e_l$, then they will charge the same price. There are two possible scenarios: (A4) Both suppliers sell to Manufacturer H and Manufacturer L, and (A5) Both suppliers sell to Manufacturer H. In Case A4, Supplier H will use a price-matching policy of $U_{Ll} = U_{Lh}$. The best strategy of Supplier L is charging p_l such that $U_{Ll} = 0$. Thus, solving $U_{Ll} = U_{Lh} = 0$ gives

$$p_{l4} = p_{h4} = V(1 - \lambda + \lambda e_l), \tag{13}$$

$$\pi_{l4} = \pi_{h4} = 2V(1 - \lambda + \lambda e_l) - ce_l^2. \tag{14}$$

In Case A5, Supplier H will use a price-matching policy of $U_{Hl} = U_{Hh}$. The best strategy of Supplier L is charging p_l such that $U_{Hl} = 0$. Thus, solving $U_{Hl} = U_{Hh} = 0$ gives

$$p_{l5} = p_{h5} = V\theta(1 - \lambda + \lambda e_l), \tag{15}$$

$$\pi_{l5} = \pi_{h5} = V\theta(1 - \lambda + \lambda e_l) - ce_l^2. \tag{16}$$

PROPOSITION 1. *When quality ratings are provided to manufacturers in the first period, then given e_h and e_l with $e_h > e_l$, the equilibrim prices are given as follows.*

(a) $1 < \theta < 3$: If $e_l \le \bar{e}_{l1}$, equilibrium prices are given by Equations (1) and (2) [Case A1]. Otherwise, equilibrium prices are given by Equations (5) and (6) [Case A2].
Here $\bar{e}_{l1} = [\lambda e_h(2\theta - 3) - (1 - \lambda)]/[2\lambda(\theta - 1)]$.
(b) $\theta \ge 3$: If $e_l \le \bar{e}_{l2}$, equilibrium prices are given by Equations (1) and (2) [Case A1]. Otherwise, equilibrium prices are given by Equations (9) and (10) [Case A3].
Here $\bar{e}_{l2} = [\theta\lambda e_h - (1 - \lambda)(\theta - 2)]/[2\lambda(\theta - 1)]$.
Given e_h and e_l with $e_h = e_l$, the equilibrim prices are given as follows.
(c) If $\theta < 2$, then equilibrium prices are given by Equation (13) [Case A4].
(d) If $\theta \ge 2$, then equilibrium prices are given by Equation (15) [Case A5].

The detailed proofs are provided in the online appendix. Intuitively, if Supplier L and Supplier H are sufficiently differentiated, then they should target different customers. The result of Proposition 3 is consistent with this intuition. When e_l is sufficiently small and sufficiently different from e_h, then equilibrium is

Case A1, where Supplier H sells to the high-type manufacturer whereas Supplier L sells to the low-type manufacturer. However, if Supplier L and Supplier H are not sufficiently differentiated, they may sell to the same type of manufacturer. In Case A2, Supplier H and Supplier L share the market of the low-type manufacturer. In Case A3 they share the market of the high-type manufacturer. The value of θ measures the high-type manufacturer's willingness-to-pay for a unit of supplier effort and 1 is the willingness-to-pay of the low-type manufacturer. If such willingness-to-pay of the high-type manufacturer is much higher than that of the low-type manufacturer ($\theta > 2$), then only the high-type manufacturers is valuable to both suppliers. Thus, Supplier H and Supplier L only sell to the high-type manufacturer (Case A3). In contrast, when $\theta \le 2$, then Supplier H will sell to both types of manufacturers. Supplier L only sells to the low-type manufacturer (Case A2).

PROPOSITION 2. *When quality ratings are provided to manufacturers in the first period, the optimal efforts of suppliers are as follows.*

(a) $1 < \theta < 2$: if $c < \hat{c}_1$, then $e_h^* = e_l^* = 1$; if $\hat{c}_1 \le c < \hat{c}_2$, then $e_h^* = 1$, $e_l^* = V\lambda/(2c)$; if $c \ge \hat{c}_2$, then $e_h^* = \left[3V\lambda + 2\sqrt{2V(c - c\lambda + V\lambda^2)}\right]/(2c)$, $e_l^* = V\lambda/(2c)$.

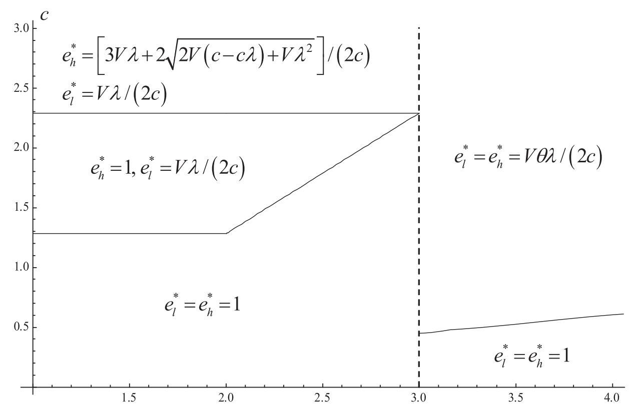
(b) $2 \le \theta < 3$: if $c < \hat{c}_3$, then $e_h^* = e_l^* = 1$; if $\hat{c}_3 \le c < \hat{c}_2$, then $e_h^* = 1$, $e_l^* = V\lambda/(2c)$; if $c \ge \hat{c}_2$, then $e_h^* = \left[3V\lambda + 2\sqrt{2V(c - c\lambda + V\lambda^2)}\right]/(2c)$, $e_l^* = V\lambda/(2c)$.

(c) $3 \le \theta$: if $c < \hat{c}_4$, then $e_h^* = e_l^* = 1$; if $c \ge \hat{c}_4$, then $e_h^* = e_l^* = V\theta\lambda/(2c)$.

where $\hat{c}_1 = \frac{1}{2}V\left(1 + \lambda + \sqrt{1 + 2\lambda}\right)$, $\hat{c}_2 = \frac{1}{2}V\left(2 + \lambda + 2\sqrt{1 + \lambda}\right)$, $\hat{c}_3 = \frac{1}{2}V\left(\theta - 1 + \lambda + \sqrt{(\theta - 1)(\theta - 1 + 2\lambda)}\right)$, $\hat{c}_4 = V\theta\lambda/2$.

Given $(V,\lambda) = (1,0.3)$, Figure 3 shows results of Proposition 2. Note that we assume that both suppliers may use a price-matching technique (Zhang 1995) to prevent their customers from being poached by the competitor. We use case A2 as an example to show the implication of this assumption. In Case A2, $U_{Lh} = U_{Ll} = 0$, Manufacturer L obtains the same net surplus from Supplier L and Supplier H. Thus, Manufacturer L will randomly buy from either Supplier L or Supplier H. The expected demand for Supplier L is 1/2 in Period 1 and Period 2. If Supplier L does not use any price matching technique, then Supplier H may cut its price by ε ($\varepsilon > 0$, $\varepsilon \to 0$) such that $U_{Lh} = \varepsilon > U_{Ll} = 0$. Now, Manufacturer L obtains higher net surplus from Supplier H than from

Figure 3 Equilibrium Efforts in (c,θ) Space When (V,λ) = (1,0.3)



Supplier L. Thus, Manufacturer L will always buy from Supplier H. It follows that the expected demand for Supplier L is zero. That is, Supplier L is competed out of the market. However, when Supplier L uses a price-matching policy "$U_{Lh} = U_{Ll}$" ($p_l \ge 0$), Supplier H is unable to poach Supplier L's customer by simply cutting its price a little bit. The reason is that when Supplier H reduces its price by ε, then Supplier L's price is automatically reduced by ε according to its price-matching policy $U_{Lh} = U_{Ll}$. This means that Manufacturer L always obtains the same net surplus from Supplier L and Supplier H. The expected demand for Supplier L is always 1/2 as long as $p_l \ge 0$. Thus, if Supplier H wants to drive Supplier L out of the market, then Supplier H must charge a price such that both types of manufacturers would choose Supplier H even though Supplier L charges $p_l = 0$.

Next, we show that Supplier H does not have any incentive to drive Supplier L out of the market in equilibrium. If that happened, then Supplier H must charge a price such that $U_{Lh} \ge U_{Ll}|_{p_l=0}$, $U_{Hh} \ge U_{Hl}|_{p_l=0}$. This leads to $p_h \le \min\left[V\theta\lambda\left(e_h^* - e_l^*\right), V\lambda\left(e_h^* - e_l^*\right)\right] = V\lambda\left(e_h^* - e_l^*\right)$. If Supplier H charges $p_h = V\lambda\left(e_h^* - e_l^*\right)$, it will obtain a profit of $\pi_h = 4p_h - c\left(e_h^*\right)^2 = 4V\lambda\left(e_h^* - e_l^*\right) - c\left(e_h^*\right)^2$. However, it can be shown that $\pi_h^* - \pi_h > 0$ always holds (see section 7.3 in the online Appendix for a detailed analysis). Therefore, two suppliers share the market in equilibrium as shown in Proposition 2.

It can also be seen that when $1 < \theta < 3$, the optimal quality levels of both Supplier L and Supplier H are not affected by θ, the taste parameter of Manufacturer H. The reason is that when $1 < \theta < 3$, the difference between two types of manufacturers is so small that both types of manufacturers are valuable to suppliers. Manufacturer L is the target customer of both suppliers. Manufacturer H has higher willingness-to-pay for a unit increase of quality than Manufacturer L. As long as Manufacturer L can afford buying from Supplier L and Supplier H, Manufacturer H must be able to afford buying from Supplier L and Supplier H.

Thus, both suppliers only need to consider the willingness-to-pay of Manufacturer L when they choose quality levels. Therefore, Manufacturer H's taste for quality (θ) does not affect the optimal quality levels of both suppliers. When $\theta \geq 3$, the difference between the two types of manufacturers is so big that Manufacturer H is much more valuable than Manufacturer L to both suppliers. Then both suppliers give up Manufacturer L and sell to Manufacturer H only. Thus, Manufacturer H's taste for quality (θ) can affect the optimal quality levels of both suppliers when the cost of developing quality is sufficiently large.

Consider the effects of the manufacturer's proportional loss from a failure of the supplier's product (λ). Intuitively, both suppliers should enhance their quality levels when manufacturers face a higher potential proportional loss. This can be seen from $de_l^*/d\lambda > 0$ and $de_h^*/d\lambda > 0$.

4. Competition with Manufacturer Assessments

In this section, we examine the case of competition where professional quality ratings are not provided in Period 1. Manufacturers must rely on quality information provided by vendors—gathered through the manufacturers' own assessment. We focus on the rational expectations equilibrium (Jerath et al. 2010, Muth 1961), where manufacturers form expectations on quality levels of suppliers based on available information, and the expectations are unbiased. That is, $E(e_l) = e_l^*$ and $E(e_h) = e_h^*$.

The quality levels of suppliers remain unknown to manufacturers in Period 1. There are two possible equilibrium outcomes in Period 1: (a) a separating equilibrium where both suppliers truly announce their types (high-quality or low-quality) and charge different prices, and (b) a pooling equilibrium where the low-quality supplier mimics the high-quality supplier by charging the same price as that charged by the high-quality supplier. It can be shown that given e_l and e_h, the low-quality supplier always has an incentive to mimic the high-quality one in Period 1 (see section 7.5 in the online Appendix for a detailed analysis). In Period 2, manufacturers know the quality levels of both suppliers. Thus, it does not make any sense for Supplier L to mimic Supplier H in Period 2. We further assume that manufacturers make *rational* decisions instead of *emotional* decisions. Otherwise, they might never buy from the low-quality supplier, even though their net surplus of buying from the low-quality supplier is greater than that of buying from the high-quality supplier in Period 2.

Thus, both suppliers appear identical to manufacturers, and they charge the same introductory price p_i. In Period 1, manufacturers randomly choose a supplier with a probability of 50% for each. We use U_{tr} to denote the net surplus of a type-t (t = H,L) manufacturer who randomly chooses a supplier.

$$U_{Lr} = \frac{1}{2}E[U_{Ll}] + \frac{1}{2}E[U_{Lh}],$$
$$U_{Hr} = \frac{1}{2}E[U_{Hl}] + \frac{1}{2}E[U_{Hh}], \tag{17}$$

where $E[U_{Ll}] = E(e_l)(V - p_i) + (1 - E(e_l))[(1 - \lambda)V - p_i]$, $E[U_{Lh}] = E(e_h)(V - p_i) + (1 - E(e_h))[(1 - \lambda)V - p_i]$, $E[U_{Hl}] = E(e_l)(\theta V - p_i) + (1 - E(e_l))[(1 - \lambda)\theta V - p_i]$, $E[U_{Hh}] = E(e_h)(\theta V - p_i) + (1 - E(e_h))[(1 - \lambda)\theta V - p_i]$.

There are two possible scenarios in equilibrium: (S1) only the high-type manufacturer can afford the introductory price p_i in Period 1, and (S2) both types of manufacturers can afford p_i in Period 1.

In the first scenario (S1), the expected demand of each supplier is $\frac{1}{2}$. The introductory price p_i is set to satisfy $U_{Hr} = 0$, or $p_i = V\theta(1 - \lambda) + \frac{1}{2}\lambda\theta V[E(e_l) + E(e_h)]$. In Period 2, suppliers charge different prices (p_h and p_l) because their quality levels are revealed to manufacturers via manufacturers' risk assessments. The profits of low-quality and high-quality suppliers can be expressed as: $\pi_l = \frac{1}{2}p_i + p_l - ce_l^2$ and $\pi_h = \frac{1}{2}p_i + p_h - ce_h^2$.

In the second scenario (S2), the expected demand of each supplier is 1. The introductory price p_i satisfies $U_{Lr} = 0$, or $p_i = V(1 - \lambda) + \frac{1}{2}\lambda V[E(e_l) + E(e_h)]$. In Period 2, suppliers charge p_l and p_h, respectively when their efforts are revealed. The profits can be expressed as: $\pi_l = p_i + p_l - ce_l^2$ and $\pi_h = p_i + p_h - ce_h^2$.

PROPOSITION 3. *When a quality rating is not provided to manufacturers in the first period, the optimal efforts of suppliers are as follows.*

(a) $1 < \theta < 2$: *if* $c < \hat{c}_a$, *then* $e_h^* = e_l^* = 1$; *if* $\hat{c}_a \leq c \leq \hat{c}_b$, *then* $e_h^* = 1$, $e_l^* = V\lambda/(4c)$; *if* $c > \hat{c}_b$, *then* $e_h^* = [3V\lambda + 2\sqrt{2V(2c - 2c\lambda + V\lambda^2)}]/(4c)$, $e_l^* = V\lambda/(4c)$.
(b) $2 \leq \theta < 3$: *if* $c < \hat{c}_c$, *then* $e_h^* = e_l^* = 1$; *if* $\hat{c}_c \leq c \leq \hat{c}_b$, *then* $e_h^* = 1$, $e_l^* = V\lambda/(4c)$; *if* $c > \hat{c}_b$, *then* $e_h^* = [3V\lambda + 2\sqrt{2V(2c - 2c\lambda + V\lambda^2)}]/(4c)$, $e_l^* = V\lambda/(4c)$.
(c) $3 \leq \theta$: *if* $c \leq \hat{c}_d$, *then* $e_h^* = e_l^* = 1$; *if* $c > \hat{c}_d$, *then* $e_h^* = e_l^* = V\theta\lambda/(4c)$.

where $\hat{c}_a = \frac{1}{4}V(1 + \lambda + \sqrt{1 + 2\lambda})$, $\hat{c}_b = \frac{1}{4}V(2 + \lambda + 2\sqrt{1 + \lambda})$, $\hat{c}_c = \frac{1}{4}V(\theta - 1 + \lambda + \sqrt{(\theta - 1)(\theta - 1 + 2\lambda)})$, $\hat{c}_d = V\theta\lambda/4$.

In this section, the suppliers appear identical to manufacturers in Period 1. Thus, the suppliers cannot segment the market by selling to different types of manufacturers. Instead, the suppliers have the same chance to sell to a specific type of manufacturer. The proof of Proposition 3 shows that when the high-type manufacturer has sufficiently high willingness-to-pay for the service (θ sufficiently large, $\theta \geq 2$), the target manufacturer is the high-type manufacturer only in Period 1. Otherwise, target manufacturers are both types in Period 1.

5. Comparison: Professional Quality Ratings vs. Manufacturer Assessments

We use Case R to denote the case where professional quality ratings are provided in Period 1, and Case NR to denote the case where professional quality ratings are not provided in Period 1 (i.e., manufacturers need to perform risk assessments by themselves).

Free-riding arises in Case NR because the low-quality supplier may mimic a high-quality supplier, and then confuse manufacturers. Intuitively, the free-riding problem should reduce the high-quality supplier's incentive to invest in quality. But, it is not obvious how the low-quality supplier's quality effort is affected. There are two conflicting effects. First, the low-quality supplier appears identical to the high-quality supplier in Period 1, so it could have incentives to enhance its quality level to increase the willingness-to-pay of manufacturers. Second, the low-quality supplier still needs to maintain an appropriate differentiation with the high-quality supplier in Period 2 to avoid intense price competition after quality levels are known to manufacturers. Since free-riding reduces the high-quality supplier's effort on quality, the low-quality supplier could also reduce its effort to keep an appropriate differentiation with the high-quality supplier.

PROPOSITION 4. *When a professional quality rating is not available, free-riding reduces the quality efforts of both the high-quality supplier and the low-quality supplier. That is,* $(e_h^*|\text{ Case R}) \geq (e_h^*|\text{ Case NR})$ *and* $(e_l^*|\text{Case R}) \geq (e_l^*|\text{Case NR})$.

Proposition 4 shows that although it is possible for the low-quality supplier to enhance its quality level to gain higher revenue in Period 1, the effect of free-riding still dominates Supplier L's desire of gaining higher revenue in Period 1.

PROPOSITION 5. *The impacts of the quality rating on the profits of both suppliers are as follows.*

(a) $1 < \theta < 2$*: The quality rating benefits Supplier L when c is moderate. It also benefits Supplier H when c is moderate. The mathematical expressions of the above results are as follows. There exists a* $c_r \in (\hat{c}_1, \hat{c}_2)$ *such that when* $c \in [\hat{c}_1, c_r)$*, the quality rating benefits Supplier H* $((\pi_h^*|\text{ Case R}) > (\pi_h^*|\text{Case NR}))$. *When* $c \in [\hat{c}_a, \hat{c}_1) \cup (c_r, +\infty)$*, the quality rating hurts Supplier H* $((\pi_h^*|\text{Case R}) < (\pi_h^*|\text{Case NR}))$. *When* $c \in (0, \hat{c}_a) \cup \{c_r\}$ *, the quality rating does not afffect Supplier H's profit* $((\pi_h^*|\text{Case R}) = (\pi_h^*|\text{Case NR}))$. *There exists a* $c_s \in (\hat{c}_a, \hat{c}_b)$ *such that when* $c \in [\hat{c}_a, c_s)$*, the quality rating benefits Supplier L* $((\pi_l^*|\text{Case R}) > (\pi_l^*|\text{Case NR}))$. *When* $c \in (c_s,+\infty)$*, the quality rating hurts Supplier L. When* $c \in (0, \hat{c}_a) \cup \{c_s\}$*, quality rating does not affect Supplier L's profit.*

(b) $2 \leq \theta < 3$*: The quality rating benefits Supplier L when c is moderate. It can benefit Supplier H when c is moderate. See the proof for mathematical expressions of the above results.*

(c) $3 \leq \theta$*: The quality rating always benefits both Supplier L and Supplier H.*

It might seem intuitive that the quality rating always helps the high-quality supplier but hurts the low-quality supplier. Proposition 5 shows that this is *not always* the case. The reason is that ratings generate two effects on competition: (1) It eliminates the free-riding problem. This effect helps the two suppliers to develop higher quality products for manufacturers. Then manufacturers have higher willingness-to-pay for the products. Both suppliers are able to charge higher prices. Thus, the quality rating can benefit both suppliers. (2) It can intensify the competition in Period 1. In Case NR, when $2 \leq \theta < 3$, both suppliers sell to Manufacturer H only. The high-quality supplier can extract all the surplus from the high-type manufacturer while the low-quality supplier can charge a high price by free-riding on the high-quality supplier. However, these benefits for both suppliers are gone when the rating is provided because both suppliers need to compete for the demand of Manufacturer L in both periods when $\hat{c}_3 \leq c$. Thus, the rating (which can intensify the competition) may hurt both suppliers.

PROPOSITION 6. *The quality rating does not affect the low-type manufacturer, whose net surplus is always zero. It hurts the high-type manufacturer when* $2 \leq \theta < 3$ *and* $\hat{c}_c \leq c < \hat{c}_3$*, where* $\hat{c}_c = \frac{1}{4}V(\theta-1+\lambda+\sqrt{(\theta-1)(\theta-1+2\lambda)})$, $\hat{c}_3 = \frac{1}{2}V(\theta-1+\lambda+\sqrt{(\theta-1)(\theta-1+2\lambda)})$. *In other regions of* (c,θ)*, the quality rating benefits the high-type manufacturer.*

This result is different from Shapiro (1986), which showed that improved information *always* helps the high-type manufacturer. The reason is that Shapiro (1986) assumed that the market is fully competitive with no profit for the sellers while we do not make such an assumption. Footnote 10 of Shapiro (1986) suggested that modeling heterogeneous sellers would permit the analysis of issues not modeled in that study. The sellers in our study are ex-post heterogeneous.

Intuitively, quality rating helps the high-type manufacturer to choose the high-quality supplier, and thus benefits the high-type manufacturer. Hence, it seems quite counterintuitive that the rating can hurt the high-type manufacturer. The reasons for such a counter intuitive result are as follows. Although the quality rating encourages suppliers to enhance their quality, it also allows high-quality suppliers to charge high prices. The high-type consumer benefits from obtaining high-quality products. However, such benefit can be offset by the high price. Therefore, the quality rating can hurt the high-type manufacturer. When $2 \le \theta < 3$ and $c < \hat{c}_3$, both suppliers exert their best efforts on product quality ($e_l = e_h = 1$) if the quality rating is provided. However, both suppliers also charge the highest price for the high-type manufacturer, leaving that manufacturer's net surplus to be zero. If the quality rating is not provided, and when $2 \le \theta < 3$ and $\hat{c}_c \le c < \hat{c}_3$, Supplier L will not choose $e_l = 1$. Instead, it will choose $e_l < 1$ to free ride on Supplier H. And Supplier L will charge a low price to sell to the low-type manufacturer in Period 2. Then Supplier H will have to charge a lower price than that when the quality rating is provided. It follows that the high-type manufacturer will obtain a positive net surplus instead of zero in $\theta \in [2,3)$ and $c \in [\hat{c}_c, \hat{c}_3)$.

Proposition 5 and Proposition 6 have important managerial implications for the business model of the quality risk rating industry. In some cases currently available, the rating agencies charge suppliers to conduct the assessment and also charge manufacturers interested in obtaining the providers' ratings (the ratings are not publicly available, but rather are provided for a fee). Our results suggest that this is not a good business model under certain conditions (e.g., when both suppliers are hurt by the quality ratings). Quality ratings have a substantial effect on competition, the suppliers, and manufacturers. Quality rating agencies must understand these effects to assess the manufacturers and suppliers willingness to pay for the rating service.

Next we examine the effect of quality ratings on social welfare. We define the social welfare as a sum of consumer surplus and producer surplus. That is, the social planner does not have any special preference for either consumer surplus or producer surplus.

PROPOSITION 7. *The impacts of the quality rating on the social welfare are as follows.*

(a1) $1 < \theta < 2$ *and* $\lambda > \frac{(3+2\theta)}{2(1+\theta)^2}$*: There exist a* $c_r \in (\hat{c}_1, \hat{c}_2)$ *such that when* $c \in [\hat{c}_a, \frac{1}{4}V\lambda(3+2\theta)) \cup [\hat{c}_1, c_r)$*, the quality rating increases social welfare. When* $c \in [\frac{1}{4}V\lambda(3+2\theta), \hat{c}_1) \cup (c_r, +\infty)$*, the quality rating reduces social welfare. When* $c \in (0, \hat{c}_a) \cup \{c_r\} \cup \{\frac{1}{4}V\lambda(3+2\theta)\}$*, the quality rating does not affect social welfare.*

(a2) $1 < \theta < 2$ *and* $\lambda \le \frac{(3+2\theta)}{2(1+\theta)^2}$*: There exist a* $c_r \in (\hat{c}_1, \hat{c}_2)$ *such that when* $c \in [\hat{c}_1, c_r)$*, the quality rating increases social welfare. When* $c \in [\hat{c}_a, \hat{c}_1) \cup (c_r, +\infty)$*, the quality rating reduces social welfare. When* $c \in (0, \hat{c}_a) \cup \{c_r\}$*, the quality rating does not affect social welfare.*

(b) $2 \le \theta < 3$*: When* $c \in [\hat{c}_3, +\infty)$*, the quality rating increases social welfare. When* $c \in [\hat{c}_c, \hat{c}_3)$*, the quality rating reduces social welfare. When* $c \in (0, \hat{c}_c)$*, the quality rating does not affect social welfare.*

(c) $3 \le \theta$*: When* $c \in (\hat{c}_d, +\infty)$*, the quality rating increases social welfare. When* $c \in (0, \hat{c}_d]$*, the quality rating does not affect social welfare.*

Surprisingly, the quality rating does not always increase social welfare. The reason is that the quality rating encourages suppliers to improve their product quality. Under certain conditions, suppliers may make too much investment on product quality and the investment may not be socially optimal.

Quality risk rating is a relatively new service compared to credit rating. In 1931, credit ratings were first endorsed by the US Office of the Comptroller of the Currency (OCC), which required banks to use current market prices for all bonds on their balance sheet rated below "investment grade". In 1936, the OCC went further and restricted banks from buying bonds below "investment grade". In comparison, quality risk ratings are not officially endorsed by the US government. Proposition 5 suggests that social planners should be prudent to encourage adoption of quality risk rating through public policy initiatives because the quality rating does not always increase social welfare.

6. Conclusion

There is growing interest in many industries for vendor quality ratings that enable enterprise manufacturers to obtain risk assessments of their suppliers expeditiously. We investigate the impact of such quality rating services on manufacturers, suppliers and social welfare.

Intuitively, some may conclude that quality risk ratings should benefit the high-quality suppliers and hurt the low-quality ones. However, we find that this is not always the case—quality ratings can hurt both high-quality and low-quality suppliers. This occurs when the absence of a quality rating softens competition allowing the low-quality supplier to appear identical to the high-quality supplier. In that case, the low-quality supplier is able to charge a higher price than otherwise and the high-quality supplier is able to avoid providing a positive net surplus to the high-type manufacturer to ensure that the manufacturer does not choose the low-quality supplier. Therefore, it is possible that the quality rating can intensify competition and hurt both suppliers. On the other hand, in some cases quality ratings can benefit both suppliers. For example, in cases where the high-type manufacturer's willingness-to-pay for a unit increase of the quality is high, it is useful for both suppliers to develop high-quality products and then charge high prices for the high-type manufacturer. Since ratings encourage both suppliers to provide high-quality products by eliminating the free-riding problem, both suppliers can benefit from ratings.

Prior literature showed that improved information always benefits the high-type manufacturer (Shapiro 1986). Our model shows that quality ratings can hurt the high-type manufacturer. This is because our model captures competition between heterogeneous providers while Shapiro (1986) assumed homogeneous providers where profit is competed away. Hence, the improved information did not affect the competition in Shapiro's model. We consider a duopolist competition, where both suppliers can earn a positive profit. We find that quality ratings have subtle effects on the competition. When the rating is provided, it may encourage suppliers to provide high-quality products. However, the rating also enables suppliers to charge high prices on manufacturers. It reduces the net surplus obtained by the high-type manufacturer. Thus, the high-type manufacturer can be hurt by a quality rating.

The quality rating also has subtle effects on social welfare. We find that it does not always increase social welfare. The policy implication is that social planners should make sure that the quality rating does not reduce social welfare before they encourage the quality risk rating. Still, the above results must be interpreted within the assumptions and limitations of our analytical model. We hope future research will relax some of these assumptions and extend the model. For example, future work might consider component procurement strategies in an assemble-to-order system with quality risk associated with component suppliers (for component procurement strategies, see Fang et al. 2008). We hope that the initial results presented in this study will motivate more research in this important area.

Acknowledgments

Zach Zhizhong Zhou acknowledges the financial support of the National Natural Science Foundation of China (Grant #: 71371121) and the Shanghai Pujiang Program (Grant #: 2PJC066). Both authors thank the editors and anonymous referees for their constructive comments and suggestions.

Note

[1]To simplify the analysis and focus on the informational effects of quality risk ratings, we assume that a manufacturer sources a single product (or batch of products) from its supplier in each period. A more general assumption is letting the quantity be endogeneously determined. However, such assumption makes the model complicated and less tractable without adding significant business insight. It is likely possible to obtain results similar to those in this study in a more complicated model.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix A. Proofs.

7 Online Appendix

7.1 Proof of Proposition 1

Proof. In Case A1, Case A2, and Case A4, given Supplier H's price matching policy, Supplier L cannot poach Supplier H's customer (or increase its demand) via simply cutting its price. Thus, the price equilibrium defined by Case A1, Case A2 or Case A4 is stable as long as Supplier H uses the price matching policy described in these cases.

However, Case A3 and Case A5 are not always stable because Supplier L has another choice: selling to the low-type manufacturer. In Case A3, Supplier L may sell to the low-type manufacturer instead of the high-type manufacturer by charging a lower price p_{l1} (note that $p_{l1} < p_{l3}$). Then Supplier L obtains π_{l1} instead of π_{l3}. Thus, Case A3 is sustainable only when $\pi_{l3} > \pi_{l1}$. Following a similar logic, Case A5 is sustainable only when $\pi_{l5} > \pi_{l4}$.

$\pi_{l1} - \pi_{l3} = V(2-\theta)(1-\lambda+\lambda e_l)$, $\pi_{l4} - \pi_{l5} = V(2-\theta)(1-\lambda+\lambda e_l)$. This suggests that Case A3 and Case A5 are not sustainable in $\theta < 2$ because even though Supplier H chooses the pricing strategy of Case A3 (or Case A5), Supplier L always has an incentive to deviate to Case A1 (or Case A4). Thus, when $e_l = e_h$, the equilibrium is Case A4 in $\theta < 2$, or Case A5 in $\theta \geq 2$.

When $e_h > e_l$ and $\theta < 2$, we only need to compare Case A1 with Case A2. Since Supplier L does not deviate, we only need to compare π_{h2} with π_{h1}. $\pi_{h1} - \pi_{h2} = V[(2\theta-3)\lambda e_h - 2(\theta-1)\lambda e_l - (1-\lambda)]$. Thus, when $\theta < 2$ and $e_l \leq [\lambda e_h(2\theta-3) - (1-\lambda)]/[2\lambda(\theta-1)]$, the equilibrium is Case A1. Otherwise, the equilibrium is Case A2.

Now, consider $e_h > e_l$ and $\theta \geq 2$. $\pi_{h3} - \pi_{h2} = V(\theta-3)(1-\lambda+\lambda e_h)$. Thus, Case A2 is not equlibrium in $\theta \geq 3$; Case A3 is not equilibrium in $2 \leq \theta < 3$. When $2 \leq \theta < 3$, we only need to compare π_{h2} with π_{h1}. $\pi_{h1} - \pi_{h2} = V[(2\theta-3)\lambda e_h - 2(\theta-1)\lambda e_l - (1-\lambda)]$. Thus, when $2 \leq \theta < 3$ and $e_l \leq [\lambda e_h(2\theta-3) - (1-\lambda)]/[2\lambda(\theta-1)]$, the equilibrium is Case A1. Otherwise, the equilibrium is Case A2. When $\theta \geq 3$, we only need to compare π_{h3} with π_{h1}. $\pi_{h1} - \pi_{h3} = V[\theta\lambda e_h - (\theta-2)(1-\lambda) - 2(\theta-1)\lambda e_l]$. Thus, when $\theta \geq 3$ and $e_l \leq [\theta\lambda e_h - (1-\lambda)(\theta-2)]/[2\lambda(\theta-1)]$, the equilibrium is Case A1. Otherwise, the equilibrium is Case A3. ■

7.2 Proof of Proposition 2

Proof. We claim that Case A1 is impossible when $1 < \theta < 3$ is satisfied. According to part (a) of Proposition 1, the inequality $e_l \leq \bar{e}_{l1} (= [\lambda e_h (2\theta - 3) - (1 - \lambda)] / [2\lambda(\theta - 1)])$ should be satisfied in Case A1. However, when $1 < \theta \leq 3/2$, then $\bar{e}_{l1} < 0$ is always satisfied. Note that $0 \leq e_l \leq 1$ always holds, the inequality $e_l \leq \bar{e}_{l1}$ cannot be satisfied in $1 < \theta \leq 3/2$. Next we consider the scenario $3/2 < \theta < 3$, where $\bar{e}_{l1}$ is an increasing function of e_h. It follows that $\bar{e}_{l1} \leq \bar{e}_{l1}|_{e_h=1} = 1 - (\theta - 2) / [2\lambda(\theta - 1)]$. Further, $\bar{e}_{l1}|_{e_h=1}$ is an increasing function of θ because $d(\bar{e}_{l1}|_{e_h=1}) / d\theta = 1/ \left[2\lambda(\theta - 1)^2\right] > 0$. We have $\bar{e}_{l1} \leq \bar{e}_{l1}|_{e_h=1} < \bar{e}_{l1}|_{e_h=1,\theta=3} = 1 - \frac{1}{4\lambda} < 0$. The last inequality holds because $0 < \lambda \leq 1/2$. Again, since $0 \leq e_l \leq 1$ always holds, the inequality $e_l \leq \bar{e}_{l1}$ cannot hold in $3/2 < \theta < 3$. This means that the inequality $e_l \leq \bar{e}_{l1}$, which is the necessary condition for Case A1, does not hold in $1 < \theta < 3$. Therefore, Case A1 is impossible in $1 < \theta < 3$.

Now, we prove part (a) of Proposition 2. We show that both suppliers have no incentive to deviate from $e_h^* = e_l^* = 1$ when $c < \hat{c}_1$ is satisfied, where $\hat{c}_1 = \frac{1}{2}V\left(1 + \lambda + \sqrt{1 + 2\lambda}\right)$. When $1 < \theta < 2$ and $e_h^* = e_l^* = 1$, both suppliers sell to both types of manufacturers (Case A4, see part (c) of Proposition 1). Inserting $e_h^* = e_l^* = 1$ in eq.(14), we get the profits of both suppliers:

$$\pi_l^* = \pi_h^* = 2V - c.$$

Suppose that Supplier L deviate from $e_l^* = 1$ to $e_l' < 1$, then the competition turns to be Case A2. The reason is that given $e_h^* > e_l'$, there are only two possible cases in $1 < \theta < 2$: Case A1 and Case A2 (see part (a) of Proposition 1). And we have shown that Case A1 is impossible in $1 < \theta < 3$. Then according to eq.(5) and eq.(7), we have $\pi_l' = \pi_{l2} = V(1 - \lambda + \lambda e_l) - ce_l^2$. We want to show that $\max_{e_l', 0 < e_l' < 1} \pi_l' < 2V - c$. Solving the F.O.C. (first-order condition) $d\pi_l'/de_l' = 0$, we get $e_l'^* = V\lambda/(2c)$. There are two possibilities: (1) $V\lambda/2 < c \leq \hat{c}_1$. In this case, $0 < e_l'^* < 1$ holds. We have $\pi_l'^* = \pi_l'|_{e_l' = V\lambda/(2c)}$. Solving $\pi_l'^* - \pi_l^* = 0$, we obtain $c = \hat{c}_1$. We note that $d(\pi_l'^* - \pi_l^*) / dc = 1 - [V\lambda/(2c)]^2 > 0$ in $c > V\lambda/2$. Thus, $\pi_l'^* \leq \pi_l^*$ always holds with the equality holding at $c = \hat{c}_1$. (2) $c \leq V\lambda/2$. In this case, $e_l'^* \geq 1$ holds. However, we have assumed that Supplier L deviate from $e_l^* = 1$ to $e_l' < 1$. Thus, $\pi_l'^* = \pi_l'|_{e_l'=1-\varepsilon\ (\varepsilon>0,\varepsilon\to 0)} = V - c < 2V - c$. Therefore, when $1 < \theta < 2$ and $c \leq \hat{c}_1$, both suppliers have no incentive to deviate from $e_i^* = 1$ to $e_i < 1$ $(i = l, h)$.

Next, we show that when $1 < \theta < 2$ and $\hat{c}_1 \leq c \leq \hat{c}_2$, suppliers do not deviate from $e_h^* = 1$ and $e_l^* = V\lambda/(2c)$. This equilibrium outcome is Case A2 because Case A1 is impossible in $1 < \theta < 3$. And $e_l = e_h$, the necessary condition for Case A4, is not satisfied. Inserting optimal efforts in eq.(7) and eq.(8), we get the profits of both suppliers:

$$\pi_l^* = V(1-\lambda) + V^2\lambda^2/(4c),$$

$$\pi_h^* = 3V - c.$$

Consider the follower Supplier L. Given $e_h^* = 1$, if Supplier L chooses $e_l' < 1$, then its profit is given by eq.(7). Solving the first order condition, we get $e_l'^* = V\lambda/(2c)$. The condition $\hat{c}_1 \leq c$ ensures that $e_l'^* \leq 1$ holds. Thus, if Supplier L chooses $e_l' < 1$, its best choice is $e_l'^* = V\lambda/(2c)$, which is exactly the equilibrium quality given by Proposition 2. If Supplier L chooses $e_l' = 1$, then its profit is given by eq.(14): $\pi_l' = 2V - c$. However, the inequality $\pi_l^* \geq \pi_l'$ always holds in $\hat{c}_1 \leq c$. Therefore, Supplier L does not have any incentive to deviate from $e_l^* = V\lambda/(2c)$. Now, consider the leader Supplier H. Its profit is higher than that of Supplier L in $\hat{c}_1 \leq c \leq \hat{c}_2$ because solving $\pi_l^* = \pi_h^*$ gives $c = \hat{c}_2$ and $d(\pi_h^* - \pi_l^*)/dc = [V\lambda/(2c)]^2 - 1 < 0$ in $V\lambda/2 < \hat{c}_1 \leq c$. If Supplier H deviate from $e_h^* = 1$ to a quality level strictly less than 1 ($e_h' < 1$), then $\pi_h' > \pi_h^*$ should hold. However, given $\pi_h' > \pi_h^* > \pi_l^*$ and $e_h' < 1$, we claim that Supplier L has an incentive to deviate from e_l^* to $e_h' + \varepsilon$ ($\varepsilon > 0$, $\varepsilon \to 0$) and become a higher-quality supplier. This is because the profit of the higher-quality supplier in Case A2 is

$$\pi_h = 3V(1 - \lambda + \lambda e_h) - ce_h^2.$$

And Supplier L will be able to get $\pi_h|_{e_h=e_h'+\varepsilon} = \pi_h' > \pi_l^*$. However, if the follower Supplier L chooses to be a higher-quality supplier, then the leader Supplier H cannot obtain a profit higher than $\pi_l^* = V(1-\lambda) + V^2\lambda^2/(4c)$. This is because the profit of the lower-quality supplier in Case A2 is

$$\pi_l = V(1-\lambda+\lambda e_l) - ce_l^2,$$

which reaches its maximum π_l^* at $e_l^* = V\lambda/(2c)$. Therefore, Supplier H also has no incentive to deviate from $e_h^* = 1$ to $e_h' < 1$.

Next, we show that when $1 < \theta < 2$ and $\hat{c}_2 < c$, suppliers do not deviate from $e_h^* = \left[3V\lambda + 2\sqrt{2V(c - c\lambda + V\lambda^2)}\right]/(2c)$ and $e_l^* = V\lambda/(2c)$. Inserting e_l^* and e_h^* in eq.(7) and eq.(8), we find that Supplier H obtains the same profit as that of Supplier L.

$$\pi_h^* = \pi_l^* = V(1-\lambda) + V^2\lambda^2/(4c).$$

It can be verified that $e_h^* < 1$ in $\hat{c}_2 < c$ because $e_h^* = 1$ at $c = \hat{c}_2$ and $de_h^*/dc < 0$. Consider the follower Supplier L. As shown above, if it chooses to be a lower-quality supplier, then its best choice is choosing $e_l^* = V\lambda/(2c)$. If it chooses a quality strictly higher than e_h^*, then it will get a profit strictly lower than π_l^*. This is because the profit of the higher-quality supplier (see eq.(8)) is a decreasing function of e_h in $e_h > 3V\lambda/(2c)$. And $e_h^* > 3V\lambda/(2c)$ is satisfied. If Supplier L chooses a quality level the same as e_h^*, then it still gets a profit strictly lower than π_l^*. The reason is that given $e_l = e_h^*$, the profit of Supplier L is given by eq.(14) (see Part(c) of Proposition 1). Inserting $e_l = e_h^*$ and $e_h = e_h^*$ in eq.(14), we find that

$$\pi_l = -\frac{V\lambda\left[5cV\lambda + 4\sqrt{2}c\sqrt{V(c - c\lambda + V\lambda^2)}\right]}{4c^2} < 0.$$

Therefore, Supplier L has no incentive to deviate. Now consider Supplier H. If it is able to obtain a profit higher than π_h^* by choosing $e_h' < 1$, then following the same argument as shown in the past paragraph, the follower Supplier L has an incentive to choose $e_h' + \varepsilon$. Then the maximum profit for Supplier H cannot exceed π_h^*. If Supplier H chooses $e_h' = 1$, then Supplier L will choose $e_l^* = V\lambda/(2c)$. Since $1 > e_h^* > 3V\lambda/(2c)$, Supplier H will obtain a profit strictly lower than π_h^*. Therefore, Supplier H also has no incentive to deviate.

Following a similar proof as above, we may prove part (b) of Proposition 2. Next, we prove part (c) of Proposition 2. Firstly, we show that both suppliers have no incentive to deviate from $e_h^* = e_l^* = 1$ when $c < \hat{c}_4$ is satisfied, where $\hat{c}_4 = V\theta\lambda/2$. When $3 \le \theta$ and $e_h^* = e_l^* = 1$, both suppliers sell to the high-type manufacturer (Case A5, see part (c) of Proposition 1). Inserting $e_h^* = e_l^* = 1$ in eq.(16), we get the profits of both suppliers:

$$\pi_l^* = \pi_h^* = \theta V - c.$$

Suppose that Supplier L deviates from $e_l^* = 1$ to $e_l' < 1$, then there are two possibilities: $e_l' > \bar{e}_{l2}|_{e_h=1}$ and $e_l' \le \bar{e}_{l2}|_{e_h=1}$ (see Proposition 1). If $e_l' > \bar{e}_{l2}|_{e_h=1}$ holds, then Supplier L's profit is expressed as eq.(11). Clearly Supplier L's profit given by eq.(11) is an increasing function in $e_l \sim [0, 1]$ as long as $c < \hat{c}_4$ is satisfied. Thus, Supplier L's optimal quality level is 1, not $e_l' < 1$. Now, consider the case where $e_l' \le \bar{e}_{l2}|_{e_h=1}$ holds and Supplier L's profit is expressed as eq.(3). Clearly π_l' is an increasing function in $e_l \sim [0, 1]$ as long as $c \le V\lambda$ is satisfied. Thus, when $c \le V\lambda$, Supplier L's optimal quality level is 1, not $e_l' < 1$. Now, consider the case where $V\lambda < c < \hat{c}_4$, then Supplier L's optimal quality level is $e_l' = V\lambda/c$. And its profit is $\pi_l' = 2V(1-\lambda) + (V\lambda)^2/c$.

$$\pi_l^* - \pi_l' = \left[-c^2 + cV(\theta - 2 + 2\lambda) - V^2\lambda^2\right]/c = w_1/c,$$

where $w_1 = -c^2 + cV(\theta - 2 + 2\lambda) - V^2\lambda^2$. Clearly w_1 is a concave quadratic function of c. And we have

$$w_1|_{c=V\lambda} = V^2\lambda(\theta - 2) > 0,$$

$$w_1|_{c=V\theta\lambda/2} = \frac{1}{4}V^2\lambda(\theta - 2)\left[2\lambda + \theta(2 - \lambda)\right] > 0.$$

This means that $\pi_l^* - \pi_l' > 0$ always holds in $V\lambda < c < \hat{c}_4$. Therefore, both suppliers do not have any incentive to deviate.

Secondly, we show that both suppliers have no incentive to deviate from $e_h^* = e_l^* = V\theta\lambda/(2c)$ when $c \geq \hat{c}_4$ is satisfied. In this case,

$$\pi_l^* = \pi_h^* = V\theta(1-\lambda) + (V\theta\lambda)^2 / (4c).$$

Suppose that Supplier L deviates from e_l^* to $e_l' < e_l^*$. When $e_l' > \bar{e}_{l2}|_{e_h=e_h^*}$ holds, then Supplier L's profit is expressed as eq.(11). Solving the first order condition $d\pi_l'/de_l = 0$, we find that the optimal quality level for Supplier L is exactly e_l^*, not e_l'. When $e_l' \leq \bar{e}_{l2}|_{e_h=e_h^*}$ holds, then Supplier L's profit is expressed as eq.(3). Supplier L's optimal quality level is $e_l' = V\lambda/c$. And its profit is $\pi_l' = 2V(1-\lambda) + (V\lambda)^2/c$.

$$\pi_l^* - \pi_l' = V(\theta-2)\left[4c(1-\lambda) + V\lambda^2(\theta+2)\right]/(4c) > 0.$$

Thus, Supplier L has no incentive to deviate from e_l^* to $e_l' < e_l^*$. Suppose that Supplier L deviates from e_l^* to $e_l' > e_l^* (= e_h^*)$, then Supplier L is a higher-quality supplier while Supplier H is a lower-quality supplier with $e_l = e_h^*$ and $e_h = e_l'$. The competiton can be either Case A3 (when $e_h^* > \bar{e}_{l2}|_{e_h=e_l'}$ holds) or Case A1 (when $e_h^* \leq \bar{e}_{l2}|_{e_h=e_l'}$ holds). It is straightforward to verify that Supplier L's optimal quality level in Case A3 is e_l^*, not $e_l' > e_l^*$. If the competition turns out to be Case A1, then Supplier L's profit can be expressed as $\pi_l' = 2V\theta\lambda(e_l' - e_h^*) + 2V(1-\lambda+\lambda e_h^*) - c(e_l')^2$. Solving the first order condition for Supplier L, we find that the optimal quality level is $e_l' = V\theta\lambda/c$ (when $V\theta\lambda < c$) or $e_l' = 1$ (when $V\theta\lambda/2 < c \leq V\theta\lambda$). It is straightforward to verify that

$$\pi_l^* - \left(\pi_l'|_{e_l'=1}\right) > \pi_l^* - \left(\pi_l'|_{e_l'=V\theta\lambda/c}\right)$$

because $\pi_l'|_{e_l'=V\theta\lambda/c}$ is the global optimum. We only need to show that $\pi_l^* - \left(\pi_l'|_{e_l'=V\theta\lambda/c}\right) > 0$.

$$\pi_l^* - \left(\pi_l'|_{e_l'=V\theta\lambda/c}\right) = w_2 V/(4c),$$

where $w_2 = 4c(1-\lambda)(\theta-2) + V\theta\lambda^2(\theta-4)$. It can be shown that w_2 is an increasing function of θ because $dw_2/d\theta = 4c(1-\lambda) + 2V\lambda^2(\theta-2) > 0$. Since $\theta > 3$, $c > \hat{c}_4 = V\theta\lambda/2 > 3V\lambda/2$ and $0 < \lambda < 1/2$, we have $w_2 > w_2|_{\theta=3} = 4c(1-\lambda) - 3V\lambda^2 > 4c(1-\lambda) - 3V\lambda^2|_{c=3V\lambda/2} = 3V\lambda(2-3\lambda) > 0$. Therefore, Supplier L has no incentive to deviate from e_l^* to $e_l' > e_l^* (= e_h^*)$. ■

7.3 Supplier H Will Not Drive Supplier L Out of the Market

Proof. Given any e_l and e_h with $e_h \geq e_l$, we want to show that driving Supplier L out of the market is not the best choice for Supplier H. If H drives L out of the market, then it obtains $\pi_h^{'} = 4V\lambda(e_h - e_l) - c(e_h)^2$ in two periods. However, if Supplier H chooses the same pricing strategy as that used in Case A2, then it can obtain a profit higher than $\pi_h^{'}$. This is because Supplier H obtains $\pi_{h2} = 3V(1-\lambda+\lambda e_h) - ce_h^2$ in Case A2 (see eq.(8)). And $\pi_{h2} - \pi_h^{'} = V[3(1-\lambda) - \lambda e_h + 4\lambda e_l] > V[3(1-\lambda) - \lambda + 4\lambda e_l] = V[3 - 4\lambda + 4\lambda e_l] > 0$ (note that $\lambda \in (0, 1/2)$). Therefore, it is not the best choice for Supplier H to drive Supplier L out of the market. ■

7.4 Proof of Proposition 3

Proof. We have assume that manufacturers are rational and that their expectations on quality levels of suppliers are unbiased. It means that (1) Given $E(e_l)$ and $E(e_h)$, manufacturer expectations on quality levels of Supplier H and Supplier L, Supplier L and Supplier H choose e_l^* and e_h^*; (2) The manufacturer expectations are unbiased in equilibrium ($E(e_l) = e_l^*$, $E(e_h) = e_h^*$). We use backward induction to derive e_l^* and e_h^*.

In Period 2, manufacturers know e_l and e_h, and are able to figure out which supplier is Supplier H (or Supplier L). Thus, the optimal prices are the same as those given by Proposition 1. In Period 1, suppliers may sell to Manufacturer H only (the first scenario S1), or sell to both Manufacturer H and Manufacturer L (the second scenario S2). In the first scenario (S1), we have

$$p_i^{S1} = V\theta(1-\lambda) + \frac{1}{2}\lambda\theta V[E(e_l) + E(e_h)]. \tag{18}$$

In the second scenario (S2), we have

$$p_i^{S2} = V(1-\lambda) + \frac{1}{2}\lambda V \left[E(e_l) + E(e_h)\right]. \tag{19}$$

The revenue obtained by each supplier in Period 1 is $\frac{1}{2}p_i^{S1}$ in S1, or p_i^{S2} in S2.

$$\frac{1}{2}p_i^{S1} - p_i^{S2} = \frac{1}{4}V(\theta-2)\left[2(1-\lambda) + \lambda\left(E(e_l) + E(e_h)\right)\right].$$

Clearly, when $\theta \geq 2$, both supplires will sell to Manufacturer H only in Period 1. Otherwise, they will sell to both Manufacturer H and Manufacturer L.

Consider the optimal efforts of both suppliers. The proof of this proposition is similar to that of Proposition 2. We show the proof for the third result of part (a) and omit other simpler proof. The third result of part (a) is: if $1 < \theta < 2$ and $c > \hat{c}_b$, then $e_h^* = \left[3V\lambda + 2\sqrt{2V\left(2c - 2c\lambda + V\lambda^2\right)}\right]/(4c)$ and $e_l^* = V\lambda/(4c)$, where $\hat{c}_b = \frac{1}{4}V\left(2 + \lambda + 2\sqrt{1+\lambda}\right)$. In this case, it is staightforward to show that $\pi_l^* = \pi_h^*$.

Now, we show that the follower Supplier L does not have any incentive to deviate e_l^*. Since $1 < \theta < 2$, Supplier L obtains p_i^{S2} in Period 1. In Period 2, the optimal price of Supplier L is given by Proposition 1 because e_l and e_h have been revealed to manufacturers. In the proof of Proposition 2, we have shown that Case A1 is impossible when $1 < \theta < 3$ is satisfied. Thus, if Supplier L chooses an effort level strictly less than e_h^* ($e_l < e_h^*$), then the competition in Period 2 is described by Case A2 (see Proposition 1). The revenue of Supplier L in Period 2 is $p_{l2}/2$, where p_{l2} is given by eq.(5). Then we obtain the net profit of Supplier L as follows.

$$\pi_l = p_i^{S2} + p_{l2}/2 - ce_l^2.$$

Solving the first order condition, we get $e_l^* = V\lambda/(4c)$. Inserting $E(e_l) = e_l^*$ and $E(e_h) = e_h^*$ in

$\pi_l^* = p_i^{S2} + p_{l2}/2 - c\left(e_l^*\right)^2$, we get

$$\pi_l^* = \frac{V}{16c}\left[8c\left(1+\theta\right)\left(1-\lambda\right) + \lambda^2 V\left(1+4\theta\right) + 2\sqrt{2}\theta\lambda\sqrt{V\left(2c - 2c\lambda + V\lambda^2\right)}\right] \tag{20}$$

This means that if Supplier L chooses a quality level strictly less than e_h^*, then it should choose e_l^*. Next, we show that Supplier L has no incentive to deviate from e_l^* to $e_l' \geq e_h^*$. If Supplier L chooses $e_l' = e_h^*$, then the competition is described by Case A4 (see Proposition 1), and $\pi_l' = p_i^{S2} + p_{l4} - c\left(e_h^*\right)^2$, where $p_{l4} = V\left(1-\lambda+\lambda e_h^*\right)$ (see eq.(13)). Inserting π_l^* obtained from eq.(20) in $\pi_l^* - \pi_l'$, we get

$$\pi_l^* - \pi_l' = \frac{V}{8c}\left[4c\left(1-\lambda\right) + 3V\lambda^2 + 2\sqrt{2}\lambda\sqrt{V\left(2c - 2c\lambda + V\lambda^2\right)}\right] > 0.$$

Thus, Supplier L does not have any incentive to deviate from e_l^* to $e_l' = e_h^*$. Now, we examine if Supplier L has any incentive to deviate from e_l^* to $e_l' > e_h^*$. If it happens, then Supplier L becomes a higher-quality supplier. It will obtain $\pi_l' = p_i^{S2} + \frac{3}{2}p_{l2}' - c\left(e_l'\right)^2$, where $p_{l2}' = p_{h2}|_{e_h = e_l'} = V\left(1-\lambda+\lambda e_l'\right)$. We find that π_l' is a concave quandratic function of e_l' with $d\pi_l'/de_l' < 0$ in $e_l' > 3V\lambda/\left(4c\right)$. However, $e_h^* > 3V\lambda/\left(4c\right)$ because

$$e_h^* - 3V\lambda/\left(4c\right) = \frac{1}{\sqrt{2}c}\sqrt{V\left(2c - 2c\lambda + V\lambda^2\right)} > 0.$$

Thus, $\pi_l' < \pi_h^* = \pi_l^*$, Supplier L has no incentive to choose $e_l' > e_h^*$.

Now, consider the leader Supplier H. If it is able to obtain a profit π_h' strictly higher than π_h^* by choosing an effort level strictly less than 1 ($e_h' < 1$), then the follower Supplier L will choose to be a higher-quality supplier by choosing $e_h' + \varepsilon$ ($\varepsilon > 0$, $\varepsilon \to 0$) instead of e_l^*. This is because Supplier L will be able to obtain π_h' instead of π_l^* with $\pi_h' > \pi_h^* = \pi_l^*$. As shown above, given the fact that Supplier L becomes a higher-quality supplier, Supplier H cannot obatin a profit strictly higher than π_l^*. Thus, $\pi_h' > \pi_h^*$ with $e_h' < 1$ is impossible. If Supplier H deviates to $e_h' = 1$ instead, then the best choice for Supplier L is still e_l^*. We have

$$e_h^* - e_h'|_{e_h'=1, e_l=e_l^*} = \frac{1}{16c}\left(16c^2 - 16cV - 8cV\lambda + V^2\lambda^2\right).$$

Note that $16c^2 - 16cV - 8cV\lambda + V^2\lambda^2 > 0$ in $c > \hat{c}_b$, we may conclude that Supplier H has no incentive to deviate from e_h^* to $e_h' = 1$.

The proof for other results of this proposition are similar to those shown in the proof of Proposition 2. Also they are simpler than the proof for the third result of part (a) of this proposition. We omit them by leaving them to readers. ■

7.5 Supplier L Will Mimic Supplier H in Period 1 When Risk Ratings are Not Provided

Proof. We use Case R to denote the case where professional quality ratings are provided in Period 1, and Case NR to denote the case where professional quality ratings are not provided in Period 1.

(1) $\theta < 2$. If Supplier L mimic Supplier H, then its revenue in Period 1 is p_i^{S2} (see Proof of Proposition 3). If Supplier L does not mimic Supplier H, then its highest possible revenue in Periol 1 is $V(1-\lambda+\lambda e_l)$ (see Proposition 1 and its proof). Note that $E(e_l) = e_l$ and $E(e_h) = e_h$ in equilibrium, we have

$$p_i^{S2} - V(1-\lambda+\lambda e_l) = \frac{1}{2}V\lambda(e_h - e_l) \geq 0.$$

(2) $\theta \geq 2$. If Supplier L mimic Supplier H, then its revenue in Period 1 is $\frac{1}{2}p_i^{S1}$ (see Proof of Proposition 3). If Supplier L does not mimic Supplier H, then its highest possible revenue in Periol 1 is $\frac{1}{2}V\theta(1-\lambda+\lambda e_l)$ (see Proposition 1 and its proof). We have

$$\frac{1}{2}p_i^{S1} - \frac{1}{2}V\theta(1-\lambda+\lambda e_l) = \frac{1}{2}V\theta(1-\lambda+\lambda e_h) > 0.$$

Therefore, Supplier L always has an incentive to mimic Supplier H in Period 1. ■

7.6 Proof of Proposition 4

Proof. We use the results of Proposition 2 and Proposition 3.

(a) When $1 < \theta < 2$: We have $\hat{c}_a < \hat{c}_b < \hat{c}_1 < \hat{c}_2$ because

$$\hat{c}_1 - \hat{c}_b = \frac{V}{4} w_5(\lambda),$$

where $w_5(\lambda) = \left(\lambda - 2\sqrt{1+\lambda} + 2\sqrt{1+2\lambda}\right)$ is an increasing function of λ in $\lambda \in (0, 1/2)$ and $w_5(\lambda) = 0$ at $\lambda = 0$. Clearly, when $0 < c \leq \hat{c}_2$, we have (e_h^*|Case R)= 1 ≥(e_h^*|Case NR). When $c > \hat{c}_2$,

$$(e_h^*|\text{Case R}) - (e_h^*|\text{Case NR}) = \frac{3V\lambda - 2\sqrt{2}w_6}{4c},$$

where $w_6 = \sqrt{V\left[2c(1-\lambda) + V\lambda^2\right]} - 2\sqrt{V\left(c - c\lambda + V\lambda^2\right)}$, which is a decreasing function of c because

$$dw_6/dc = \frac{V(1-\lambda)}{\sqrt{V}}\left(\frac{1}{\sqrt{2c(1-\lambda) + V\lambda^2}} - \frac{1}{\sqrt{c(1-\lambda) + V\lambda^2}}\right) < 0.$$

Thus, we have

$$(e_h^*|\text{Case R}) - (e_h^*|\text{Case NR}) > \frac{3V\lambda - 2\sqrt{2}w_6}{4c}|_{c=\hat{c}_2} \geq 0.$$

The last inequality holds because $3V\lambda - 2\sqrt{2}w_6|_{c=\hat{c}_2} = Vw_7(\lambda)$, where $w_7(\lambda)$ is a function of λ and it can be verified that $w_7(\lambda) \geq 0$ always holds in $\lambda \in (0, 1/2)$. Now, consider the effort level of Supplier L. When $0 < c \leq \hat{c}_1$, we have (e_l^*|Case R)= 1 ≥(e_l^*|Case NR). When $c > \hat{c}_1$, we have (e_l^*|Case R)−(e_l^*|Case NR)=$V\lambda/(4c) > 0$.

(b) When $2 \leq \theta < 3$: We want to show that $\hat{c}_c < \hat{c}_b < \hat{c}_3 < \hat{c}_2$.

$$\hat{c}_3 - \hat{c}_b = \frac{V}{4} w_8(\theta, \lambda),$$

where $w_8(\theta, \lambda) = 2\theta - 4 + \lambda - 2\sqrt{1+\lambda} + 2\sqrt{(\theta-1)(\theta-1+2\lambda)}$. $w_8(\theta, \lambda)$ is an increasing function of θ because

$$dw_8(\theta, \lambda)/d\theta = 2 + \frac{2(\theta - 1 + \lambda)}{\sqrt{(\theta-1)(\theta-1+2\lambda)}} > 0.$$

Thus, $w_8(\theta,\lambda) > w_8(\theta,\lambda)|_{\theta=2} = \left(\lambda - 2\sqrt{1+\lambda} + 2\sqrt{1+2\lambda}\right) = w_5(\lambda)$. As shown above, $w_5(\lambda) \geq 0$. Therefore, we have $\hat{c}_3 - \hat{c}_b > 0$. It is straightforward to show that $\hat{c}_c < \hat{c}_b$ and $\hat{c}_3 < \hat{c}_2$. Thus, it follows that $\hat{c}_c < \hat{c}_b < \hat{c}_3 < \hat{c}_2$. Using a similar proof as shown in part (a), we may show that $(e_h^*|\text{Case R}) \geq (e_h^*|\text{Case NR})$ and $(e_l^*|\text{Case R}) \geq (e_l^*|\text{Case NR})$.

(c) When $\theta \geq 3$. It is straightforward to find that $(e_h^*|\text{Case R}) \geq (e_h^*|\text{Case NR})$ and $(e_l^*|\text{Case R}) \geq (e_l^*|\text{Case NR})$. ■

7.7 Proof of Proposition 5

Proof. (a) $1 < \theta < 2$: We have $\hat{c}_a < \hat{c}_b < \hat{c}_1 < \hat{c}_2$ (see the proof of Proposition 4). When $c < \hat{c}_a$, $(\pi_h^*|\text{Case R}) = (\pi_h^*|\text{Case NR}) = 2V - c$.

When $\hat{c}_a \leq c \leq \hat{c}_b$, $(\pi_h^*|\text{Case R}) = 2V - c$, $(\pi_h^*|\text{Case NR}) = \frac{1}{2}V(5-\lambda) + \frac{1}{8c}V^2\lambda^2 - c$.

$$(\pi_h^*|\text{Case NR}) - (\pi_h^*|\text{Case R}) = \frac{V}{8c}\left[4c(1-\lambda) + V\lambda^2\right] > 0.$$

When $\hat{c}_b < c < \hat{c}_1$, $(\pi_h^*|\text{Case R}) = 2V - c$, $(\pi_h^*|\text{Case NR}) = w_9 V/(16c)$, where $w_9 = 24c(1-\lambda) + 9V\lambda^2 + 4\sqrt{2}\lambda\sqrt{V\left[2c(1-\lambda) + V\lambda^2\right]} > 0$. Thus, $(\pi_h^*|\text{Case NR})/(\pi_h^*|\text{Case R}) = w_9 V/[16c(2V-c)]$. We want to show that $w_9 V > 16c(2V-c)$, then $(\pi_h^*|\text{Case NR}) > (\pi_h^*|\text{Case R})$. It can be verified that $w_9 V$ and $-16c(2V-c)$ are increasing functions of c in $c \in (\hat{c}_b, \hat{c}_1]$. Thus, $w_9 V - 16c(2V-c) \geq w_9 V - 16c(2V-c)|_{c=\hat{c}_b} > 0$. The last inequality holds because $w_9 V - 16c(2V-c)|_{c=\hat{c}_b}$ can be written as $V \cdot w_{10}(\lambda)$, where $w_{10}(\lambda)$ is a function of λ. And it is easy to verify that $w_{10}(\lambda) > 0$ in $\lambda \in (0, 1/2)$.

When $\hat{c}_1 \leq c \leq \hat{c}_2$, $(\pi_h^*|\text{Case R}) = 3V - c$, $(\pi_h^*|\text{Case NR}) = w_9 V/(16c)$, and $(\pi_h^*|\text{Case NR})/(\pi_h^*|\text{Case R}) = w_9 V/[16c(3V-c)]$. We want to show that $w_9 V - [16c(3V-c)]$ is an increasing function of c. We find that $d\{w_9 V - [16c(3V-c)]\}/dc > 16c + 16(c-3V) + 24V(1-\lambda) > 16c + 16(c-3V) + 24V(1-\lambda)|_{c=\hat{c}_1} > 0$. Thus, $w_9 V - [16c(3V-c)]$ is an increasing function of c. Further, it is easy to show that $w_9 V - [16c(3V-c)]|_{c=\hat{c}_1} < 0$ and $w_9 V - [16c(3V-c)]|_{c=\hat{c}_2} > 0$. Thus, there exists a $c_r \in (\hat{c}_1, \hat{c}_2)$ such that when $c \in [\hat{c}_1, c_r)$, $(\pi_h^*|\text{Case R}) > (\pi_h^*|\text{Case NR})$; when $c \in [c_r, \hat{c}_2)$, $(\pi_h^*|\text{Case R}) \leq (\pi_h^*|\text{Case NR})$.

When $\hat{c}_2 < c$, $(\pi_h^*|\text{Case R}) = V(1-\lambda) + \frac{1}{4c}V^2\lambda^2$, $(\pi_h^*|\text{Case NR}) = w_9 V/(16c)$, and $(\pi_h^*|\text{Case NR})/(\pi_h^*|\text{Case R}) = \frac{1}{4}w_9/\left[4c(1-\lambda)+V\lambda^2\right]$. We find that $w_9 - 4\left[4c(1-\lambda)+V\lambda^2\right] = 8c(1-\lambda) + 5V\lambda^2 + 4\sqrt{2}\lambda\sqrt{V\left[2c(1-\lambda)+V\lambda^2\right]} > 0$. Therefore, $(\pi_h^*|\text{Case R}) < (\pi_h^*|\text{Case NR})$ in $\hat{c}_2 < c$.

The proof for the impact of quality rating on Supplier L's profit is similar to and simplier than the proof for Supplier H. So we omit it.

(b) When $2 \leq \theta < 3$: The mathematical expressions of the part (b) results are as follows.

When $2 \leq \theta < \theta_1$, then there exists $\hat{\lambda}(\theta)$ and $c_r \in [\hat{c}_3, \hat{c}_2)$ such that when $\lambda > \hat{\lambda}(\theta)$ and $c \in [\hat{c}_3, c_r)$, the quality rating benefits Supplier H ($(\pi_h^*|\text{Case R}) > (\pi_h^*|\text{Case NR})$). Otherwise, the quality rating either hurts Supplier H or does not affect Supplier H's profit. When $\theta_1 \leq \theta \leq \theta_2$, the quality rating either hurts Supplier H or does not affect Supplier H's profit. When $\theta_2 < \theta$, then there exists $c_{r1} \in [\hat{c}_c, \hat{c}_b]$ and $c_{r2} \in [\hat{c}_b, \hat{c}_3]$ such that when $\lambda > \left(9-\theta^2\right)/\theta^2$ and $c \in (\hat{c}_{r1}, \hat{c}_{r2})$, the quality rating benefits Supplier H. Otherwise, the quality rating either hurts Supplier H or does not affect Supplier H's profit. Here $\theta_1 = 2.42313$, $\theta_2 = 2.44949$.

There exists $c_s \in [\hat{c}_c, \hat{c}_3]$ such that when $c \in [\hat{c}_c, c_s)$, the quality rating benefits Supplier L ($(\pi_l^*|\text{Case R}) > (\pi_l^*|\text{Case NR})$). Otherwise, the quality rating either hurts Supplier L or does not affect Supplier L's profit.

Next, we show the proof for the impact of quality rating on Supplier H's profit. We have $\hat{c}_c < \hat{c}_b < \hat{c}_3 < \hat{c}_2$ (see the proof of Proposition 4). To obtain the above results, we need to compare $(\pi_h^*|\text{Case R})$ with $(\pi_h^*|\text{Case NR})$ at several critical points: $\hat{c}_c$, $\hat{c}_b$, $\hat{c}_3$ and $\hat{c}_2$. Clearly, $(\pi_h^*|\text{Case R}) = (\pi_h^*|\text{Case NR})$ in $c < \hat{c}_c$. When $c = \hat{c}_c$, we have

$$(\pi_h^*|_{c=\hat{c}_c}\text{ Case NR}) = \frac{1}{4}V\left[7+\theta^2-\lambda-(1+\theta)\sqrt{(\theta-1)(\theta-1+2\lambda)}\right],$$

$$(\pi_h^*|_{c=\hat{c}_c}\text{ Case R}) = \frac{1}{4}V\left[1+3\theta-\lambda-\sqrt{(\theta-1)(\theta-1+2\lambda)}\right].$$

Note that $(\pi_h^*|_{c=\hat{c}_c}\text{ Case NR}) - (\pi_h^*|_{c=\hat{c}_c}\text{ Case R})$ is a decreasing function of λ and that $(\pi_h^*|_{c=\hat{c}_c}\text{ Case NR}) - (\pi_h^*|_{c=\hat{c}_c}\text{ Case R}) = 0$ at $\lambda = \hat{\lambda}_1 = 2\left(9-9\theta+5\theta^2-\theta^3\right)/\left[\theta^2(\theta-1)\right]$. We may conclude that when $\lambda > \hat{\lambda}_1$, then $(\pi_h^*|_{c=\hat{c}_c}$ Case NR$) < (\pi_h^*|_{c=\hat{c}_c}$ Case R$)$. Otherwise, $(\pi_h^*|_{c=\hat{c}_c}$ Case NR$) \geq (\pi_h^*|_{c=\hat{c}_c}$ Case R$)$. Note that $\hat{\lambda}_1 \geq 1/2$ in $\theta \in [2, 2.46378]$ and that $\lambda < 1/2$ should be satisfied, we claim that $(\pi_h^*|_{c=\hat{c}_c}$ Case NR$) < (\pi_h^*|_{c=\hat{c}_c}$ Case R$)$ does not hold in $\theta \in [2, 2.46378]$.

When $c = \hat{c}_b$, we have

$$(\pi_h^*|_{c=\hat{c}_b} \text{ Case NR}) = \frac{1}{4}V\left[4(1+\theta) - \lambda - 2(1+\theta)\sqrt{1+\lambda}\right],$$

$$(\pi_h^*|_{c=\hat{c}_b} \text{ Case R}) = \frac{1}{4}V\left[4\theta - 2 - \lambda - 2\sqrt{1+\lambda}\right].$$

Solving $(\pi_h^*|_{c=\hat{c}_b}$ Case NR$) - (\pi_h^*|_{c=\hat{c}_b}$ Case R$) = 0$, we get $\lambda = \hat{\lambda}_2 = \left(9-\theta^2\right)/\theta^2$. Note that $(\pi_h^*|_{c=\hat{c}_b}$ Case NR$) - (\pi_h^*|_{c=\hat{c}_b}$ Case R$)$ is a decreasing function of λ, we claim that when $\lambda > \hat{\lambda}_2$, then $(\pi_h^*|_{c=\hat{c}_b}$ Case NR$) < (\pi_h^*|_{c=\hat{c}_b}$ Case R$)$. Otherwise, $(\pi_h^*|_{c=\hat{c}_b}$ Case NR$) \geq (\pi_h^*|_{c=\hat{c}_b}$ Case R$)$. Note that $\hat{\lambda}_2 \geq 1/2$ in $\theta \in [2, \theta_2]$, where $\theta_2 = 2.44949$. Since $\lambda < 1/2$ should be satisfied, we claim that $(\pi_h^*|_{c=\hat{c}_b}$ Case NR$) < (\pi_h^*|_{c=\hat{c}_b}$ Case R$)$ does not hold in $\theta \in [2, \theta_2]$.

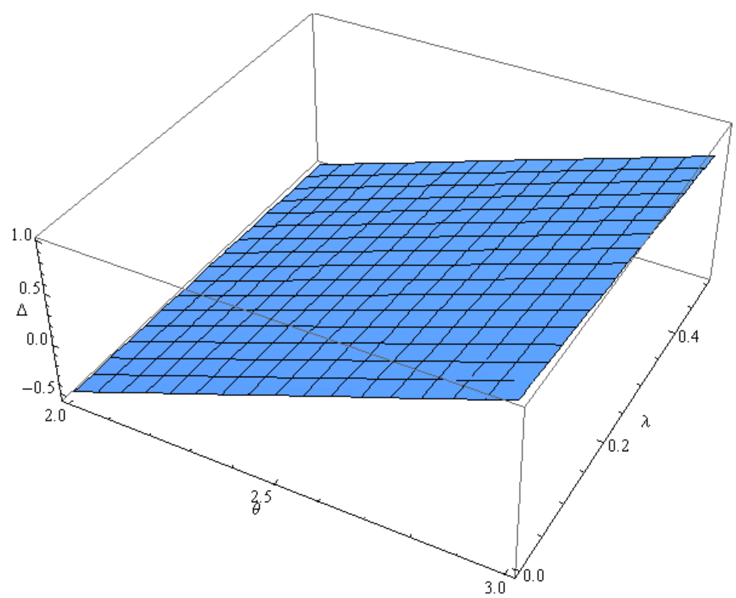

Figure 4: $(\pi_h^*|_{c=\hat{c}_3}$ Case NR$)/V - (\pi_h^*|_{c=\hat{c}_3}$ Case R$)/V$

When $c = \hat{c}_3$, we have

$$\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right)/V = \frac{1}{2}(1+\theta)(1-\lambda) + \frac{\lambda}{8w_{10}}\left[\lambda(1+4\theta) + 2\sqrt{2}\theta\sqrt{\lambda^2 + (1-\lambda)w_{10}}\right],$$

$$\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right)/V = \frac{1}{2}\left(7-\theta-\lambda-\sqrt{(\theta-1)(\theta-1+2\lambda)}\right).$$

where $w_{10} = \theta - 1 + \lambda + \sqrt{(\theta-1)(\theta-1+2\lambda)}$. Now, we examine the sign of $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right)/V - \left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right)/V$ in $\theta \in [2,3)$ and $\lambda \in (0,1/2)$. We may use the numerical method to find conditions under which $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right) - \left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right) < 0$ holds. According to the numerical analysis, when $\theta \geq \theta_1$ with $\theta_1 = 2.42313$, $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right) - \left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right) \geq 0$ holds in $\lambda \in (0,1/2)$. When $\theta < \theta_1$, then there exists a $\hat{\lambda}(\theta)$ such that when $\hat{\lambda}(\theta) < \lambda < 1/2$, $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right) - \left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right) < 0$. Otherwise, $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case NR}\right) - \left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right) \geq 0$.

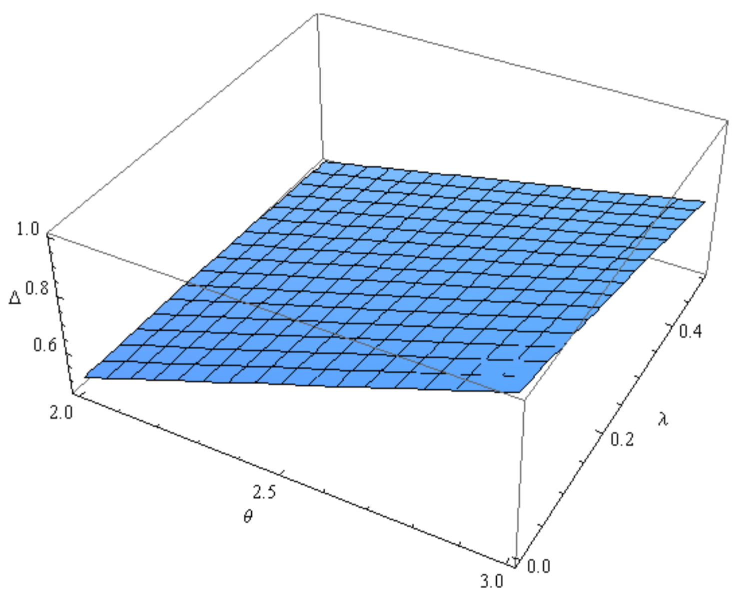

Figure 5: $\left(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case NR}\right)/V - \left(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case R}\right)/V$

Since $\left(\pi_h^*|_{c=\hat{c}_3} \text{ Case R}\right)$ is not a continuous function at $c = \hat{c}_3$, we need to compare $\left(\pi_h^*|\text{Case NR}\right)$ with $\left(\pi_h^*|\text{Case R}\right)$ at $c = \hat{c}_3 - \varepsilon$ $(\varepsilon > 0, \varepsilon \to 0)$. When $c = \hat{c}_3 - \varepsilon$, we have

$$\left(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case NR}\right) = \frac{V}{2}(1+\theta)(1-\lambda) + \frac{\lambda V}{8w_{10}}\left[\lambda(1+4\theta) + 2\sqrt{2}\theta\sqrt{\lambda^2 + (1-\lambda)w_{10}}\right],$$

$$\left(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case R}\right) = \frac{V}{2}\left(1+\theta-\lambda-\sqrt{(\theta-1)(\theta-1+2\lambda)}\right).$$

We may use numerical analysis to show that $(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case NR})-(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case R}) > 0$ holds in $\theta \in [2,3)$ and $\lambda \in (0,1/2)$. The numerical analysis shows that $(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case NR})-(\pi_h^*|_{c=\hat{c}_3-\varepsilon} \text{ Case R})$ reaches the minimum at $(\theta,\lambda) = (2,1/2)$. And the minimum is $0.462129 > 0$.

Using a similar numerical analysis as shown above, we may show that $(\pi_h^*|_{c=\hat{c}_2} \text{ Case NR}) - (\pi_h^*|_{c=\hat{c}_2} \text{ Case R}) > 0$ holds in $\theta \in [2,3)$ and $\lambda \in (0,1/2)$.

Next, we compare $d(\pi_h^*|\text{Case NR})/dc$ with $d(\pi_h^*|\text{Case R})/dc$ in $[\hat{c}_c,\hat{c}_b]$, $(\hat{c}_b,\hat{c}_3)$, $[\hat{c}_3,\hat{c}_2]$ and $(\hat{c}_2,+\infty)$. When $c \in [\hat{c}_c,\hat{c}_b]$, we have

$$d(\pi_h^*|\text{Case R})/dc = -1,$$

$$d(\pi_h^*|\text{Case NR})/dc = -1 - \frac{V^2\theta\lambda^2}{16c^2}.$$

Clearly $d(\pi_h^*|\text{Case R})/dc > d(\pi_h^*|\text{Case NR})/dc$. This means that if $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c = \hat{c}_b$, then there exists $c_{r1} \in [\hat{c}_c,\hat{c}_b]$ such that when $c \in (\hat{c}_{r1},\hat{c}_b]$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ holds. Otherwise, if $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ at $c = \hat{c}_b$, then $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ holds in $c \in [\hat{c}_c,\hat{c}_b]$.

When $c \in (\hat{c}_b,\hat{c}_3)$, we have

$$d(\pi_h^*|\text{Case R})/dc = -1,$$

$$d(\pi_h^*|\text{Case NR})/dc = -\frac{V^2\lambda^2(1+4\theta)}{16c^2} - \frac{2\sqrt{2}V^2\lambda\theta\left(c-c\lambda+V\lambda^2\right)}{16c^2\sqrt{V^2\lambda^2+2cV(1-\lambda)}}.$$

We want to show that $d(\pi_h^*|\text{Case NR})/dc > -1$. That is, although both $(\pi_h^*|\text{Case R})$ and $(\pi_h^*|\text{Case NR})$ are decreasing functions of c, the slope of $(\pi_h^*|\text{Case R})$ is steeper than that of $(\pi_h^*|\text{Case NR})$. It is straightforward to verify that $\partial^2(\pi_h^*|\text{Case NR})/\partial c\partial\theta < 0$, meaning that $d(\pi_h^*|\text{Case NR})/dc \geq d(\pi_h^*|\text{Case NR})/dc|_{\theta=3}$. Given $\theta = 3$, the other constraint $c \in (\hat{c}_b,\hat{c}_3)$ can be further simplified to $\min(\hat{c}_b|_{\theta=3}) < c < \max(\hat{c}_3|_{\theta=3})$, which is equivalent to $V < c < \left(\frac{5}{4}+\sqrt{\frac{3}{2}}\right)V$. Let $c = \gamma V$ with $\gamma \in \left(1,\frac{5}{4}+\sqrt{\frac{3}{2}}\right)$ and insert $c = \gamma V$ in $d(\pi_h^*|\text{Case NR})/dc|_{\theta=3}$, we obtain $d(\pi_h^*|\text{Case NR})/dc|_{\theta=3} =$

$g(\gamma,\lambda)$, a function of γ and λ with $\gamma \in \left(1, \frac{5}{4}+\sqrt{\frac{3}{2}}\right)$ and $\lambda \in (0, 1/2)$. Using the numerical method to analyze $g(\gamma,\lambda)$ in $\gamma \in \left(1, \frac{5}{4}+\sqrt{\frac{3}{2}}\right)$ and $\lambda \in (0,1/2)$, we find that $g(\gamma,\lambda) > g(1,1/2) = -\frac{1}{320}\left(-65-18\sqrt{10}\right) = -0.381003 > -1$. Thus, $d(\pi_h^*|\text{Case NR})/dc \geq d(\pi_h^*|\text{Case NR})/dc|_{\theta=3} > -1$. We have shown that $(\pi_h^*|_{c=\hat{c}_3-\varepsilon}\text{ Case NR}) > (\pi_h^*|_{c=\hat{c}_3-\varepsilon}\text{ Case R})$. Therefore, if $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c=\hat{c}_b$, then there exists $c_{r2} \in [\hat{c}_b, \hat{c}_3]$ such that when $c \in [\hat{c}_b, \hat{c}_{r2})$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ holds. Otherwise, if $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ at $c=\hat{c}_b$, then $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ holds in $c \in (\hat{c}_b, \hat{c}_3)$.

When $c \in [\hat{c}_3, \hat{c}_2]$, we have

$$d(\pi_h^*|\text{Case R})/dc = -1,$$

$$d(\pi_h^*|\text{Case NR})/dc = -\frac{V^2\lambda^2(1+4\theta)}{16c^2} - \frac{2\sqrt{2}V^2\lambda\theta\left(c-c\lambda+V\lambda^2\right)}{16c^2\sqrt{V^2\lambda^2+2cV(1-\lambda)}}.$$

Clearly, $d(\pi_h^*|\text{Case R})$ and $d(\pi_h^*|\text{Case NR})/dc$ are the same as those in $c \in (\hat{c}_b, \hat{c}_3)$. Thus, $d(\pi_h^*|\text{Case NR})/dc > -1$. Note that we have shown that $(\pi_h^*|_{c=\hat{c}_2}\text{ Case NR}) > (\pi_h^*|_{c=\hat{c}_2}\text{ Case R})$. Therefore, if $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c=\hat{c}_3$, then there exists $c_r \in [\hat{c}_3, \hat{c}_2)$ such that when $c \in [\hat{c}_3, c_r)$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ holds. Otherwise, if $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ at $c=\hat{c}_3$, then $(\pi_h^*|\text{Case NR}) \geq (\pi_h^*|\text{Case R})$ holds in $c \in [\hat{c}_3, \hat{c}_2]$.

When $c \in (\hat{c}_2, +\infty)$, using a similar numerical analysis as shown above, we may show that $d(\pi_h^*|\text{Case R})/dc > d(\pi_h^*|\text{Case NR})/dc$. The slope of $(\pi_h^*|\text{Case NR})$ is steeper than that of $(\pi_h^*|\text{Case R})$. We have shown that $(\pi_h^*|_{c=\hat{c}_2}\text{ Case NR}) > (\pi_h^*|_{c=\hat{c}_2}\text{ Case R})$. Further, $\lim_{c\to+\infty}(\pi_h^*|\text{Case NR}) - (\pi_h^*|\text{Case R}) = \frac{V}{2}(1-\lambda)(\theta-1) > 0$. Thus, $(\pi_h^*|\text{Case NR}) > (\pi_h^*|\text{Case R})$ is satisfied in $c \in (\hat{c}_2, +\infty)$. Otherwise, we cannot have $\lim_{c\to+\infty}(\pi_h^*|\text{Case NR}) - (\pi_h^*|\text{Case R}) > 0$.

To summarize the above results. We list the following major results.

(1) If $\theta_2 < \theta$ and $\lambda > \left(9-\theta^2\right)/\theta^2$, then $(\pi_h^*|_{c=\hat{c}_b}\text{ Case NR}) < (\pi_h^*|_{c=\hat{c}_b}\text{ Case R})$.

(2) If $\theta < \theta_1$ and $\hat{\lambda}(\theta) < \lambda$, then $(\pi_h^*|_{c=\hat{c}_3}\text{ Case NR}) < (\pi_h^*|_{c=\hat{c}_3}\text{ Case R})$.

(3) If $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c=\hat{c}_b$, then there exists $c_{r1} \in [\hat{c}_c, \hat{c}_b]$ such that when $c \in (\hat{c}_{r1}, \hat{c}_b]$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$.

(4) If $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c=\hat{c}_b$, then there exists $c_{r2} \in [\hat{c}_b, \hat{c}_3]$ such that when

$c \in [\hat{c}_b, \hat{c}_{r2})$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$.

(5) If $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$ at $c = \hat{c}_3$, then there exists $c_r \in [\hat{c}_3, \hat{c}_2)$ such that when $c \in [\hat{c}_3, c_r)$, $(\pi_h^*|\text{Case NR}) < (\pi_h^*|\text{Case R})$.

Combining these results together, we may get the result of the impact of quality rating on Supplier H.

The proof for the impact of quality rating on Supplier L's profit is similar to and simplier than the proof for Supplier H. We omit it by leaving it to readers.

(c) The proof of part (c) is simplier than the proof of part (a) and (b). Thus, we omit it. ■

7.8 Proof of Proposition 6

Proof. We denote consumer surplus by CS.

(a) $1 < \theta < 2$: We have $\hat{c}_a < \hat{c}_b < \hat{c}_1 < \hat{c}_2$ (see the proof of Proposition 4). When $c < \hat{c}_a$, $(CS|\text{Case R}) = 2U_{Hh} = 2e_h(\theta V - p_h) + 2(1-e_h)[\theta(1-\lambda)V - p_h]$, where $e_h = 1$, $p_h = V$; $(CS|\text{Case NR}) = U_{Hr} + U_{Hh} = \frac{1}{2}V\theta\lambda[E(e_h) + E(e_l)] + V\theta(1-\lambda) - p_i + e_h(\theta V - p_h) + (1-e_h)[\theta(1-\lambda)V - p_h]$, where $E(e_h) = 1$, $E(e_l) = 1$, $p_i = V$, $p_h = V$, $e_h = 1$. Thus, $(CS|\text{Case R}) = (CS|\text{Case NR}) = 2V(\theta - 1)$.

When $\hat{c}_a \le c \le \hat{c}_2$, it can be shown that $(CS|\text{Case R}) = 2V(\theta-1)$. And it can be verified that $d[(CS|\text{Case NR})]/dc < 0$, $d[(CS|\text{Case R})]/dc = 0$. Note that $(CS|_{c=\hat{c}_a}\text{ Case R}) = 2V(\theta-1)$, $(CS|_{c=\hat{c}_a}\text{ Case NR}) = \frac{1}{2}\left(5 - \sqrt{1+2\lambda}\right)V(\theta-1)$, we have $(CS|_{c=\hat{c}_a}\text{ Case R}) > (CS|_{c=\hat{c}_a}\text{ Case NR})$. It follows that $(CS|\text{Case R}) > (CS|\text{Case NR})$ in $\hat{c}_a \le c < \hat{c}_2$.

When $\hat{c}_2 < c$, we have

$$(CS|\text{Case R}) - (CS|\text{Case NR}) = \frac{V\lambda(\theta-1)}{4c^2}(7cV\lambda + w_{11}),$$

where $w_{11} = 8\sqrt{2}c\sqrt{V(c - c\lambda + V\lambda^2)} - 3\sqrt{2}c\sqrt{V(2c - 2c\lambda + V\lambda^2)}$. It can be verified that $w_{11} > 0$. Thus, $(CS|\text{Case R}) > (CS|\text{Case NR})$ in $\hat{c}_2 < c$.

(b) $2 \le \theta < 3$, We have $\hat{c}_c < \hat{c}_b < \hat{c}_3 < \hat{c}_2$ (see the proof of Proposition 4).

When $c < \hat{c}_a$, it can be verified that $(CS|\text{Case R}) = (CS|\text{Case NR}) = 0$.

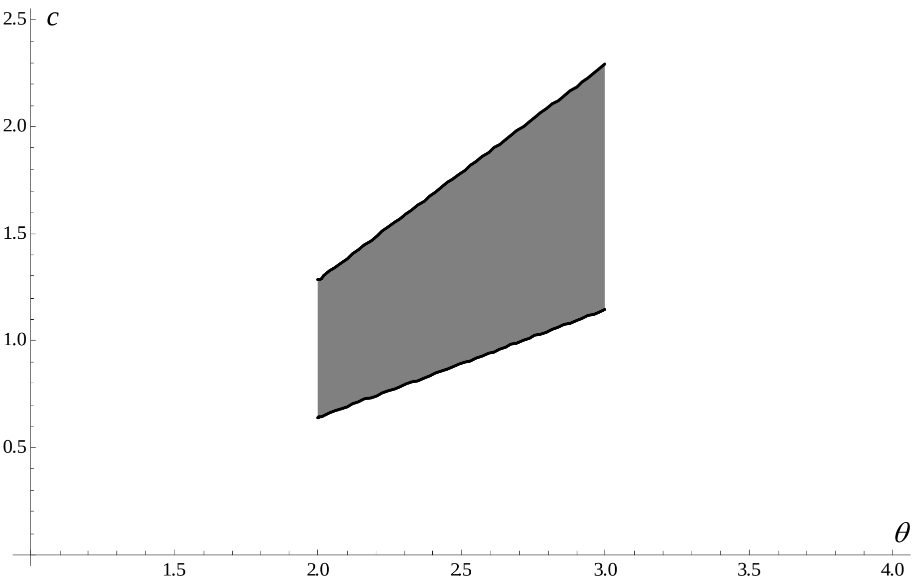

Figure 6: Given (V, λ)=(1,0.3), the quality rating hurts the high-type manufacturer in the shaded region.

When $\hat{c}_c \leq c < \hat{c}_3$, $(CS|\text{Case R}) = 0$. However, it can be shown that $(CS|\text{Case NR}) > 0$. Thus, $(CS|\text{Case NR}) > (CS|\text{Case R})$ in this $c-$region.

When $\hat{c}_3 \leq c \leq \hat{c}_2$, $(CS|\text{Case R}) = 2V(\theta - 1)$. $(CS|\text{Case NR})$ is a decreasing function in $\hat{c}_b < c \leq \hat{c}_2$. And we have $(CS|_{c=\hat{c}_b} \text{ Case NR}) = V(\theta - 1) < 2V(\theta - 1)$. Thus, $(CS|\text{Case R}) > (CS|\text{Case NR})$ in this $c-$region.

When $\hat{c}_2 < c$

$$(CS|\text{Case R}) = \frac{V(\theta-1)}{c}\left[2c(1-\lambda) + 3V\lambda^2 + 2\sqrt{2}\lambda\sqrt{V\left(c - c\lambda + V\lambda^2\right)}\right],$$

$$(CS|\text{Case NR}) = \frac{V(\theta-1)}{4c}\left[4c(1-\lambda) + 3V\lambda^2 + 2\sqrt{2}\lambda\sqrt{V\left(2c - 2c\lambda + V\lambda^2\right)}\right].$$

It is straightforward to verify that $(CS|\text{Case R}) > (CS|\text{Case NR})$.

(c) $3 \leq \theta$. $(CS|\text{Case R}) = (CS|\text{Case NR}) = 0$. ■

7.9 Proof of Proposition 7

Proof. We denote social welfare by SW.

(a1) $1 < \theta < 2$ and and $\lambda > \frac{(3+2\theta)}{2(1+\theta)^2}$: We have $\hat{c}_a < \hat{c}_b < \hat{c}_1 < \hat{c}_2$ (see the proof of Proposition 4).

When $c < \hat{c}_a$, it is easy to show that $(SW|\text{Case R}) = (SW|\text{Case NR}) = 2\left(V + V\theta - c\right).$

When $\hat{c}_a \leq c \leq \hat{c}_b$, $(SW|\text{Case R}) = 2\left(V + V\theta - c\right)$, $(SW|\text{Case NR}) = (CS|\text{Case NR}) + \pi_l + \pi_h = 2V\left(1+\theta\right) - c - \frac{1}{2}V\lambda\left(2+\theta\right) + \frac{V^2\lambda^2}{16c}\left(3+2\theta\right)$.

Thus,

$$(SW|\text{Case R}) - (SW|\text{Case NR}) = \frac{1}{16c}\left(4c - V\lambda\right)\left[\left(3+2\theta\right)V\lambda - 4c\right]$$

We claim that $(4c - V\lambda) > 0$ in $\hat{c}_a \leq c \leq \hat{c}_b$ because $4c - V\lambda|_{c=\hat{c}_a} = V\left(1+\sqrt{1+2\lambda}\right) > 0$ and $4c - V\lambda|_{c=\hat{c}_b} = 2V\left(1+\sqrt{1+\lambda}\right) > 0$. Now, consider the sign of $\left(3+2\theta\right)V\lambda - 4c$. We find that $\left(3+2\theta\right)V\lambda - 4c|_{c=\hat{c}_a} = V\left[2\lambda\left(1+\theta\right) - 1 - \sqrt{1+2\lambda}\right] > 0$. The last inequality holds because $V\left[2\lambda\left(1+\theta\right) - 1 - \sqrt{1+2\lambda}\right]$ is an increasing function of λ in $\lambda > \frac{(3+2\theta)}{2(1+\theta)^2}$, and $V\left[2\lambda\left(1+\theta\right) - 1 - \sqrt{1+2\lambda}\right]|_{\lambda=\frac{(3+2\theta)}{2(1+\theta)^2}} = 0$. We also find that $\left(3+2\theta\right)V\lambda - 4c|_{c=\hat{c}_b} = 2V\left[\lambda\theta - 1 + \lambda - \sqrt{1+\lambda}\right] < 0$. The last inequality holds because $2V\left[\lambda\theta - 1 + \lambda - \sqrt{1+\lambda}\right]$ is an increasing function of λ, $\lambda < 1/2$, and $2V\left[\lambda\theta - 1 + \lambda - \sqrt{1+\lambda}\right]|_{\lambda=\frac{1}{2}} = \frac{\theta}{2} - 1.72474 < 0$ (note that $1 < \theta < 2$). Therefore, $c = \frac{1}{4}V\lambda\left(3+2\theta\right)$ is the only one root of $(SW|\text{Case R}) - (SW|\text{Case NR}) = 0$ in $\hat{c}_a \leq c \leq \hat{c}_b$. When $\hat{c}_a \leq c < \frac{1}{4}V\lambda\left(3+2\theta\right)$, $(SW|\text{Case R}) > (SW|\text{Case NR})$. When $\frac{1}{4}V\lambda\left(3+2\theta\right) < c \leq \hat{c}_b$, $(SW|\text{Case R}) < (SW|\text{Case NR})$. When $c = \frac{1}{4}V\lambda\left(3+2\theta\right)$, $(SW|\text{Case R}) = (SW|\text{Case NR})$.

When $\hat{c}_b < c < \hat{c}_1$, we have

$$(SW|\text{Case R}) = 2V\left(\theta+1\right) - 2c,$$

$$(SW|\text{Case NR}) = V\left(1-\lambda\right)\left(1+2\theta\right) + \frac{V^2\lambda^2}{8c}\left(10\theta-1\right) + \frac{\sqrt{2}\lambda V}{4c}\left(3\theta-1\right)\sqrt{V\left[2c\left(1-\lambda\right)+V\lambda^2\right]}. \tag{21}$$

Let $\Delta_{SW} = (SW|\text{Case NR}) - (SW|\text{Case R})$. We claim that $\Delta_{SW} > 0$ in $\hat{c}_b < c < \hat{c}_1$. To show it, we follow three steps: (1) showing that $\Delta_{SW} > 0$ at $c = \hat{c}_b$, and (2) showing that

$d\left[(SW|\text{Case NR})\right]/dc > d\left(SW|\text{Case R}\right)/dc$.

Since Δ_{SW} is continuous at $c=\hat{c}_b$, it is straightforward to show that $\Delta_{SW} > 0$ at $c=\hat{c}_b$ because we have shown it in the case of $\hat{c}_a \leq c \leq \hat{c}_b$.

$d\left(SW|\text{Case R}\right)/dc = -2$. $d\left[(SW|\text{Case NR})\right]/dc$ is a decreasing function of θ. Thus, $d\left[(SW|\text{Case NR})\right]/dc > d\left[(SW|\text{Case NR})\right]/dc|_{\theta=2}$. We find that $d\left[(SW|\text{Case NR})\right]/dc|_{\theta=2}$ is an increasing function of c. Thus, $d\left[(SW|\text{Case NR})\right]/dc|_{\theta=2} > d\left[(SW|\text{Case NR})\right]/dc|_{\theta=2,c=\hat{c}_b}$. It is easy to verify that $d\left[(SW|\text{Case NR})\right]/dc|_{\theta=2,c=\hat{c}_b} > -2$.

When $\hat{c}_1 \leq c \leq \hat{c}_2$,

$$(SW|\text{Case R}) = \frac{V^2\lambda^2}{4c} - c + V\left(2+2\theta-\lambda\right),$$

and $(SW|\text{Case NR})$ is given by eq.(21). Let $\Delta_{SW} = (SW|\text{Case NR}) - (SW|\text{Case R})$. We want to show that (1) $\Delta_{SW} < 0$ at $c=\hat{c}_1$, (2) $\Delta_{SW} > 0$ at $c=\hat{c}_2$, and (3) $d\left[(SW|\text{Case NR})\right]/dc > d\left(SW|\text{Case R}\right)/dc$ in $\hat{c}_1 \leq c \leq \hat{c}_2$. Then there must exist a $c_r \in (\hat{c}_1, \hat{c}_2)$ such that $\Delta_{SW} < 0$ in $c_r \in [\hat{c}_1, c_r)$, and $\Delta_{SW} > 0$ in $c_r \in (c_r, \hat{c}_2]$.

It can be verified that $\Delta_{SW}|_{c=\hat{c}_1}$ is a decreasing function of θ. Thus, $\Delta_{SW}|_{c=\hat{c}_1} \leq \Delta_{SW}|_{c=\hat{c}_1,\theta=2}$. It can be shown that the sign of $\Delta_{SW}|_{c=\hat{c}_1,\theta=2}$ is the same as the sign of $f_1(\lambda) = -4+\lambda-4\tau+4\sqrt{2+2\tau-2\lambda\tau}$, where $\tau=\sqrt{1+2\lambda}$. It is straightforward to verify that $f_1(\lambda) < 0$ in $0<\lambda<1/2$. Therefore, $\Delta_{SW}|_{c=\hat{c}_1} < 0$.

We find that $\Delta_{SW}|_{c=\hat{c}_2}$ is also a decreasing function of θ. Thus, $\Delta_{SW}|_{c=\hat{c}_2} > \Delta_{SW}|_{c=\hat{c}_2,\theta=1}$. The sign of $\Delta_{SW}|_{c=\hat{c}_2,\theta=1}$ is the same as the sign of $f_2(\lambda) = 8+8\tau(1-\lambda)-\lambda(4-\lambda)+4\lambda\sqrt{4-2\lambda+4\tau-4\lambda\tau}$, where $\tau=\sqrt{1+\lambda}$. It is straightforward to verify that $f_2(\lambda) > 0$ in $0<\lambda<1/2$. Therefore, $\Delta_{SW}|_{c=\hat{c}_2} > 0$.

Let $\Delta D_{SW} = d\left[(SW|\text{Case NR})\right]/dc - d\left(SW|\text{Case R}\right)/dc$. We find that ΔD_{SW} is a decreasing function of θ. Thus, $\Delta D_{SW} > \Delta D_{SW}|_{\theta=1}$. Insert $c=\gamma V$ in $\Delta D_{SW}|_{\theta=1}$, we find that the sign of $\Delta D_{SW}|_{\theta=1}$ is determined by $f(\gamma,\lambda)$, where

$$f_1(\gamma,\lambda) = 40\gamma^2(1-\lambda)+16\gamma\lambda^2-\lambda(1-\lambda)\left[7\lambda+4\sqrt{2\lambda^2+4\gamma(1-\lambda)}\right].$$

Since $\hat{c}_1 \leq c \leq \hat{c}_2$ is equivalent to $\frac{1}{2}V\left(1+\lambda+\sqrt{1+2\lambda}\right) \leq c \leq \frac{1}{2}V\left(2+\lambda+2\sqrt{1+\lambda}\right)$, we have $\frac{1}{2}\left(1+\lambda+\sqrt{1+2\lambda}\right) \leq \gamma \leq \frac{1}{2}\left(2+\lambda+2\sqrt{1+\lambda}\right)$. Note that $0<\lambda<1/2$, we have $1<\gamma<\frac{5}{4}+\sqrt{\frac{3}{2}}$. Using the numerical method to find the minimum of $f_1(\gamma,\lambda)$ in $\lambda \in [0,1/2]$ and $\gamma \in \left[1, \frac{5}{4}+\sqrt{\frac{3}{2}}\right]$, we find that the mimimum of $f_1(\gamma,\lambda)$ is $f_1(\gamma,\lambda)|_{\gamma=1,\lambda=1/2} = 21.544 > 0$. Therefore, $d\left[(SW|\text{Case NR})\right]/dc > d\left(SW|\text{Case R}\right)/dc$ in $\hat{c}_1 \leq c \leq \hat{c}_2$. Therefore, we may conclude that there exists a $c_r \in (\hat{c}_1, \hat{c}_2)$ such that $\Delta_{SW} < 0$ in $c_r \in [\hat{c}_1, c_r)$, and $\Delta_{SW} > 0$ in $c_r \in (c_r, \hat{c}_2]$.

When $\hat{c}_2 < c$,

$$(SW|\text{Case R}) = \frac{V^2\lambda^2}{4c} - c + V\left(2+2\theta-\lambda\right),$$

and $(SW|\text{Case NR})$ is given by eq.(21). Let $\Delta_{SW} = (SW|\text{Case NR}) - (SW|\text{Case R})$. Since $\hat{c}_2 = \frac{1}{2}V\left(2+\lambda+2\sqrt{1+\lambda}\right)$ and $0<\lambda<1/2$, the inequality $\hat{c}_2 < c$ means that $2V < c$. Let $c = V/\tau$, then $\hat{c}_2 < c$ can be expressed as $0 < \tau < 1/2$. We want to show that Δ_{SW} is a decreasing function of θ. Inserting $c = V/\tau$ in $d\Delta_{SW}/d\theta$, we find that the sign of $d\Delta_{SW}/d\theta$ is the same as the sign of $f_2(\gamma,\lambda) = -7\lambda + 3\sqrt{\left[4-2\lambda\left(2-\gamma\lambda\right)\right]/\gamma} - 8\sqrt{2}\sqrt{\left[1-\lambda\left(1-\gamma\lambda\right)\right]/\gamma}$. Using the numerical method, we find that the maximum of $f_2(\gamma,\lambda)$ is -7.51472 at $(\lambda,\gamma) = (0,1/2)$. Thus, $d\Delta_{SW}/d\theta < 0$. It follows that $\Delta_{SW} > \Delta_{SW}|_{\theta=1}$. Inserting $c = V/\tau$ in $\Delta_{SW}|_{\theta=1}$, we find that the sign of $\Delta_{SW}|_{\theta=1}$ is the same as the sign of $f_3(\gamma,\lambda) = 8(1-\lambda) + 5\gamma\lambda^2 + 4\gamma\lambda\sqrt{\left[4-2\lambda\left(2-\gamma\lambda\right)\right]/\gamma}$. Using the numerical method, we find that the minimum of $f_3(\gamma,\lambda)$ is 4 at $(\lambda,\gamma) = (1/2,0)$. Thus, $\Delta_{SW} > 0$ holds in $\hat{c}_2 < c$.

The proof for part(a2), part(b) and part(c) is simplier than that for part(a1). Thus, we omit it. ■